Self-Localisation and Route Learning in Mobile Robots Through System Identification

Ulrich Nehmzow
Department of Computer Science
University of Essex

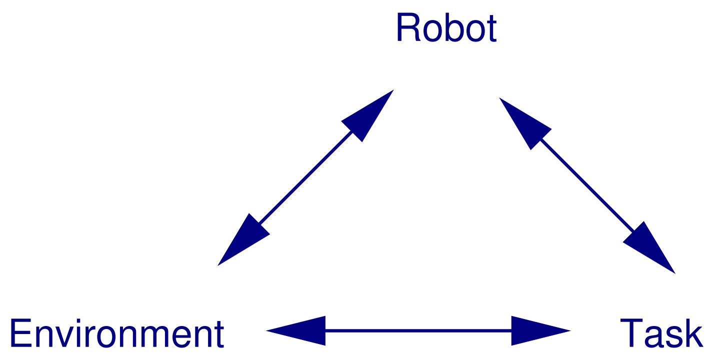
Analytical
& Cognitive
Robotics

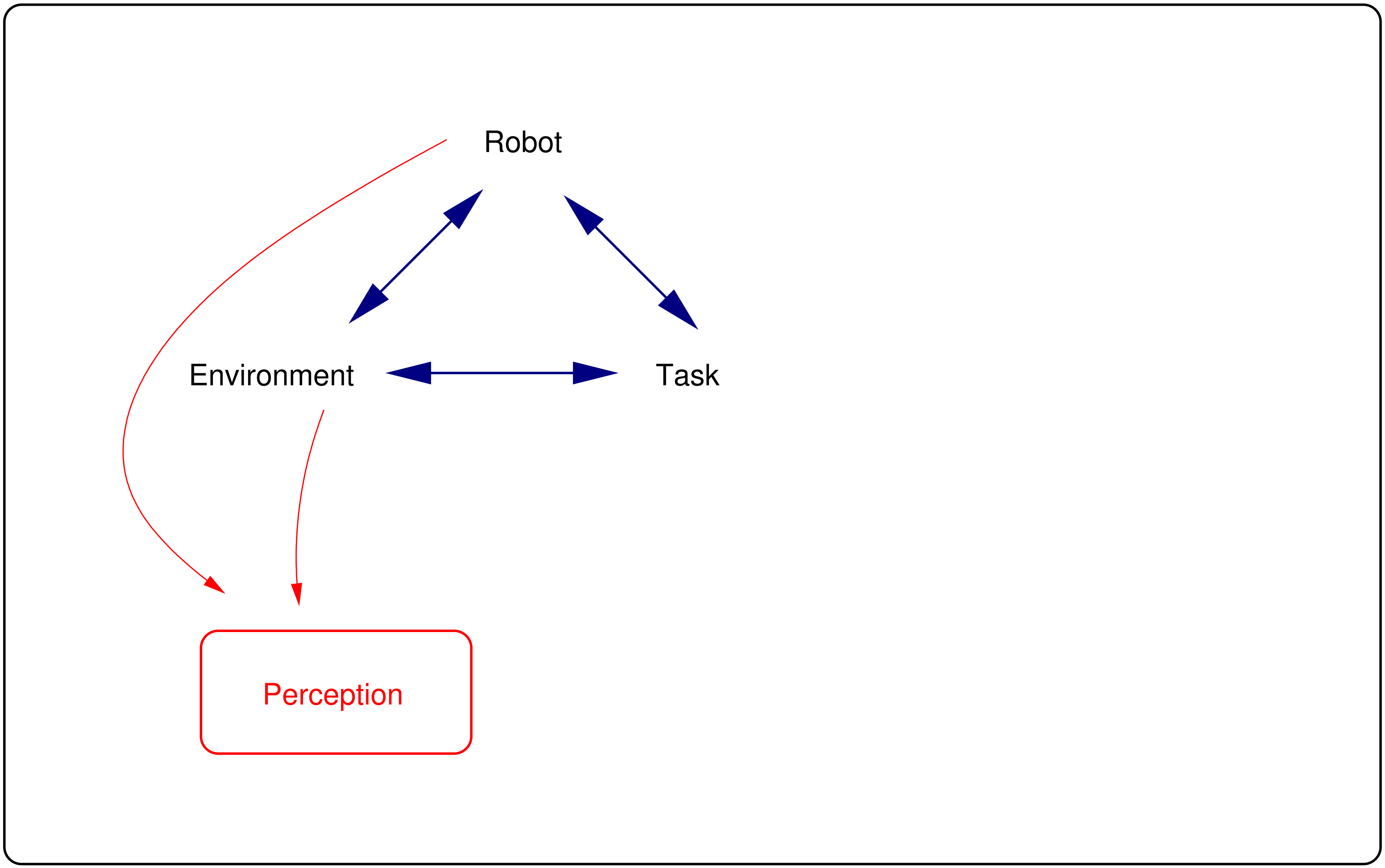
Acknowledgements

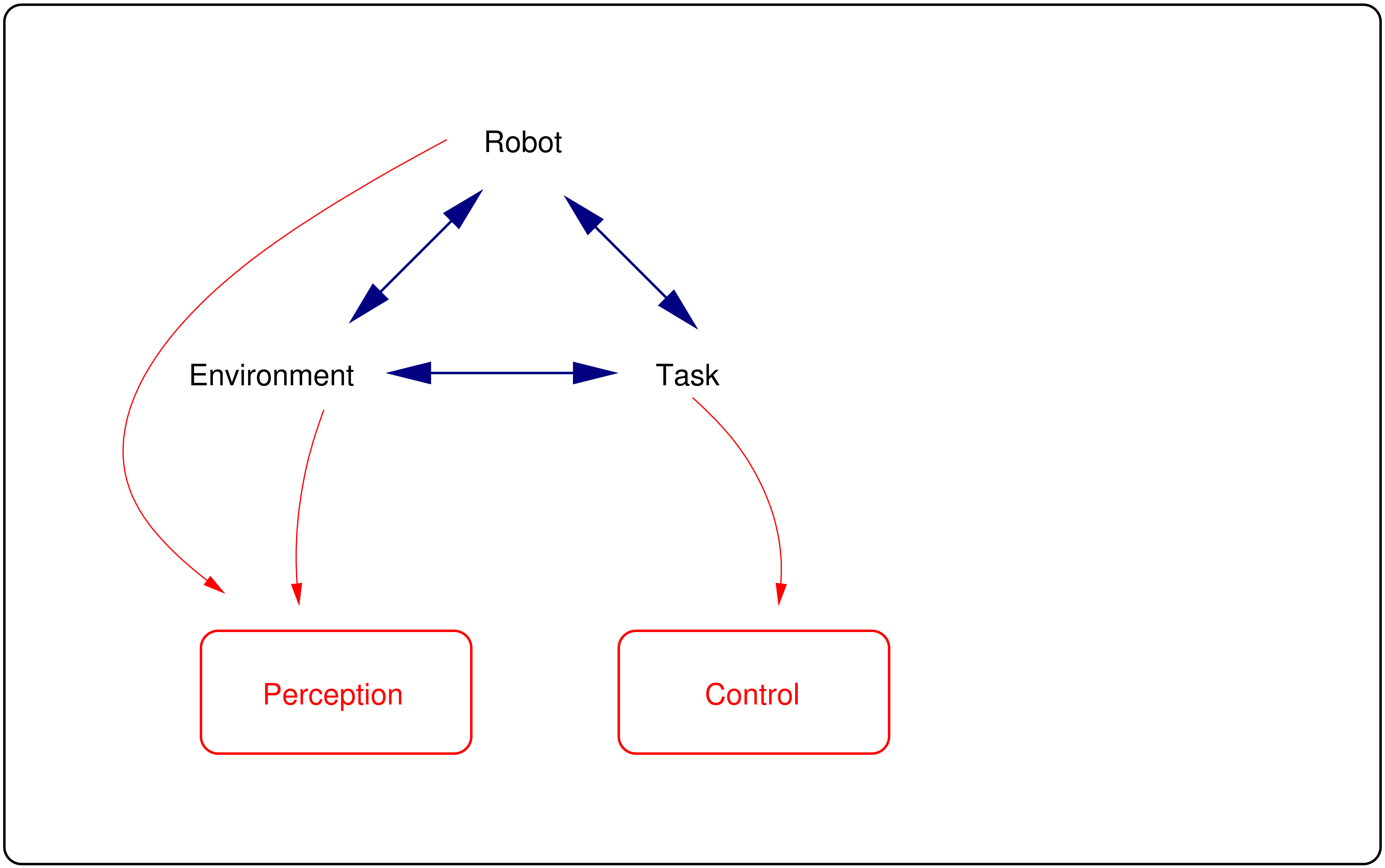
The RobotMODIC Team:

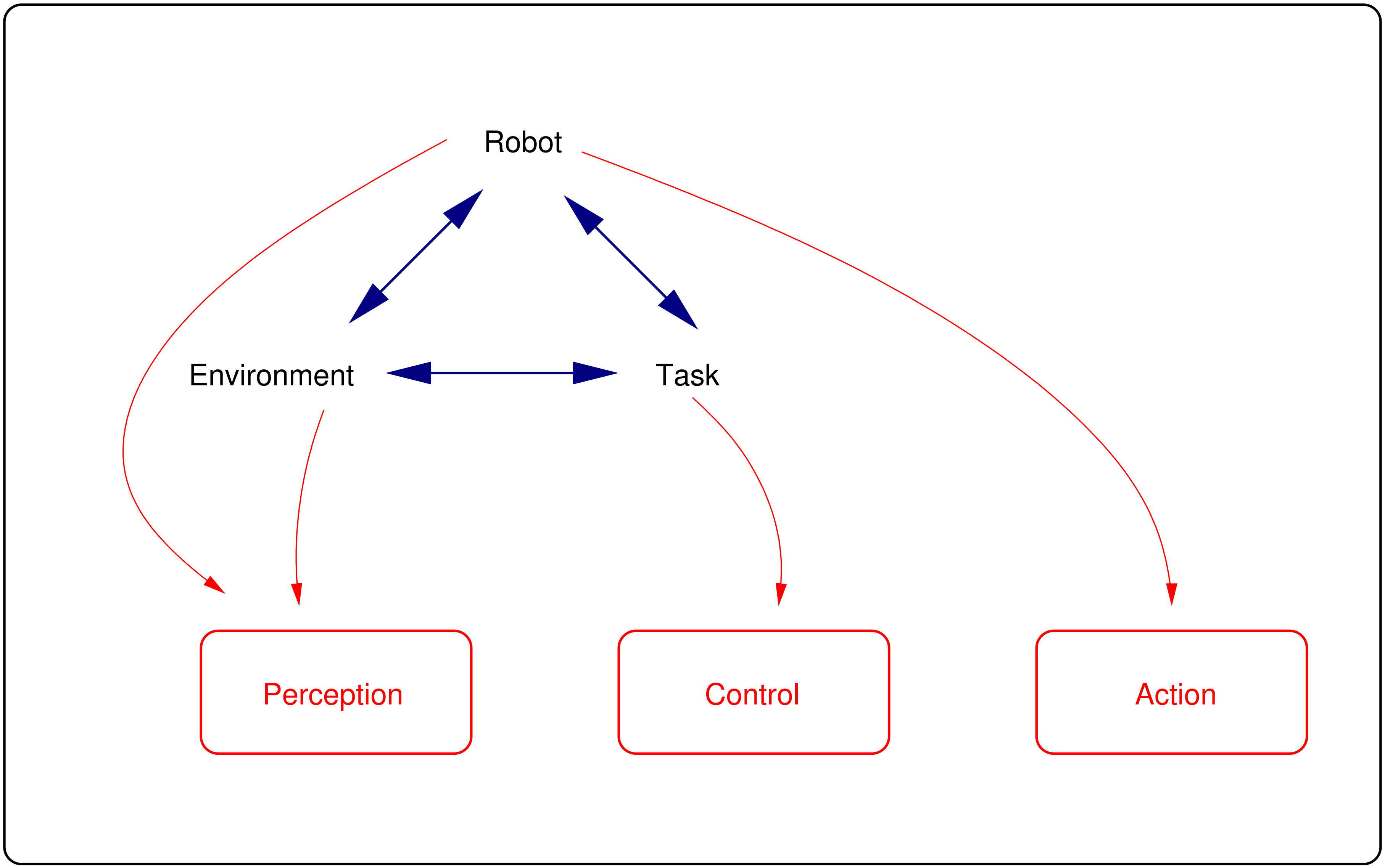
Theocharis Kyriacou, Roberto Iglesias and Steve Billings

Robot-Environment Interaction




Robot
Environment
Task
Perception


Robot
Environment
Task
Perception
Control



Robot
Environment
Task
Perception
Control
Action

Objectives of the RobotMODIC Project

RobotMODIC: Robot Modelling, Identification and Characterisation

1. Theory: coherent body of hypothetical, conceptual and pragmatic generalisations and principles that forms the general frame of reference within which mobile robotics research is conducted.

   - Formulation of testable hypotheses
   - Allow predictions
   - Essential for “off-line” design.

Objectives of the RobotMODIC Project

RobotMODIC: Robot Modelling, Identification and Characterisation

1. Theory: coherent body of hypothetical, conceptual and pragmatic generalisations and principles that forms the general frame of reference within which mobile robotics research is conducted.
   - Formulation of testable hypotheses
   - Allow predictions
   - Essential for "off-line" design.
2. Modelling of robot-environment interaction

Objectives of the RobotMODIC Project

RobotMODIC: Robot Modelling, Identification and Characterisation

1. Theory: coherent body of hypothetical, conceptual and pragmatic generalisations and principles that forms the general frame of reference within which mobile robotics research is conducted.
   - Formulation of testable hypotheses
   - Allow predictions
   - Essential for "off-line" design.
2. Modelling of robot-environment interaction
3. Identification and characterisation of robot-environment interaction

Objectives of the RobotMODIC Project

RobotMODIC: Robot Modelling, Identification and Characterisation

1. Theory: coherent body of hypothetical, conceptual and pragmatic generalisations and principles that forms the general frame of reference within which mobile robotics research is conducted.
   - Formulation of testable hypotheses
   - Allow predictions
   - Essential for “off-line” design.
2. Modelling of robot-environment interaction
3. Identification and characterisation of robot-environment interaction
4. Tools

Possible Approaches for Investigating Robot Behaviour

- Measurement and analysis

Possible Approaches for Investigating Robot Behaviour

- Measurement and analysis

- Obtaining simplified models that retain the essential properties of the robot

Experimental Setup

- Experimentation with *Magellan Pro* mobile robot
- Perception and action data is logged at 5Hz

Experimental Setup

- Experimentation with *Magellan Pro* mobile robot

- Perception and action data is logged at 5Hz

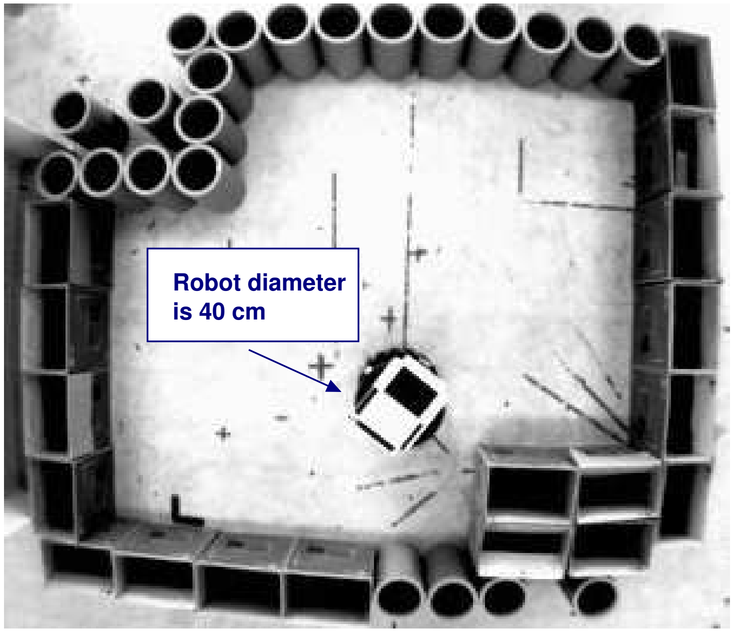
- Experimental Scenarios

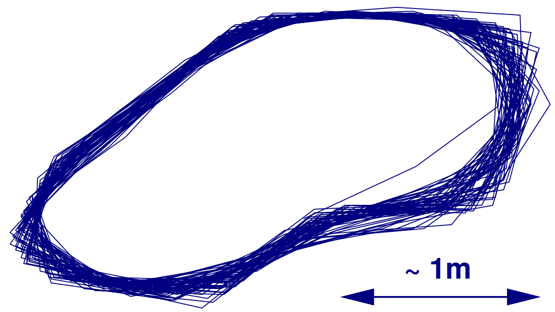
  1. Perception-based self localisation (subsymbolic)

  2. Perception-based, user-supervised route learning (again, subsymbolic)

- Objective: To obtain a quantitative, analysable representation of robot-environment interaction in closed mathematical form

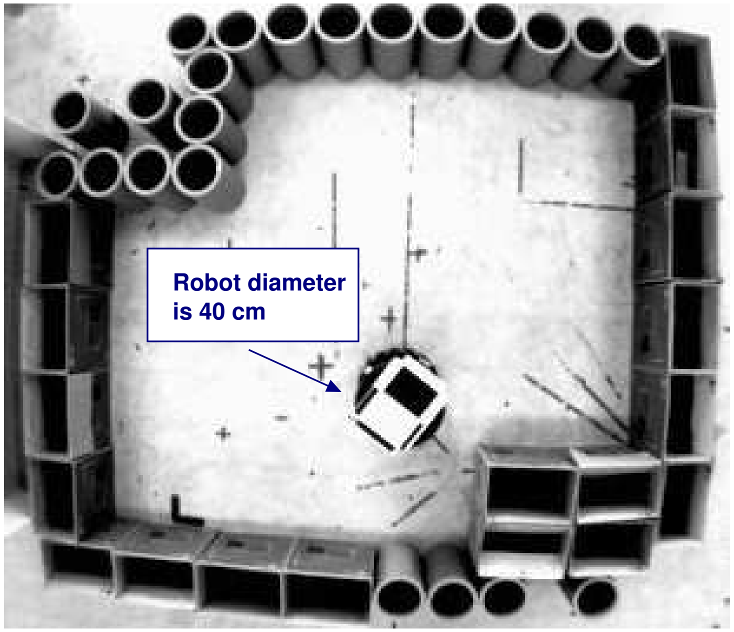
Experiment 1: Perception-Based Localisation

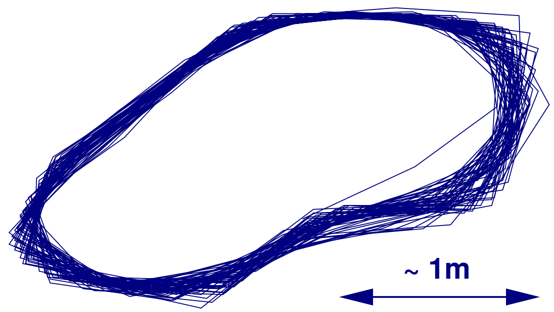




Position and perception sampled every 6.25 s

Experiment 1: Perception-Based Localisation





Position and perception sampled every 6.25 s

Interlude: Obtaining Transparent Models Through ARMAX and NARMAX

Goal: to obtain a model of the form

$y(k) = F[y(k-1), y(k-2), ..., y(k-n_y), u(k-d), ..., u(k-d-n_\mu), e(k-1), \ldots, e(k-n_e)] + e(k)$

where $y(k)$ is the sampled output, $u(k)$ the input and $e(k)$ noise.

n_y, n_u, n_e, are the orders, and d is a time delay.

F[ ] is a (nonlinear) function and is typically taken to be a polynomial.

- ARMAX: (Linear) auto-regressive, moving average model with exogeneous inputs.
  Readily available in packages such as Scilab or Matlab.

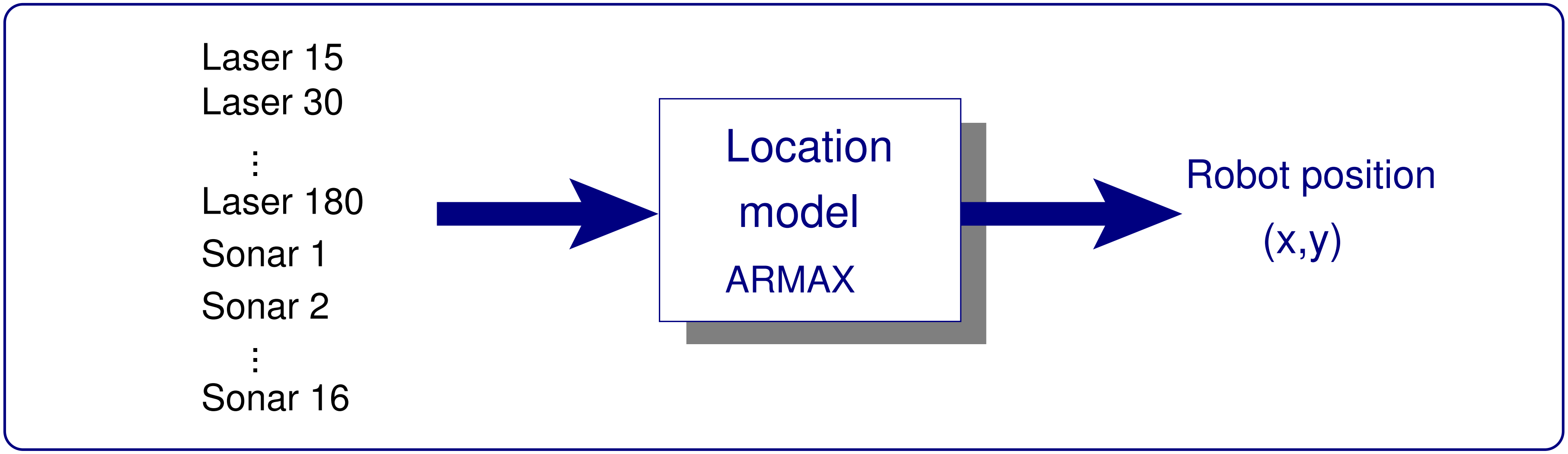
- ARMAX: (Linear) auto-regressive, moving average model with exogeneous inputs.
  Readily available in packages such as Scilab or Matlab.

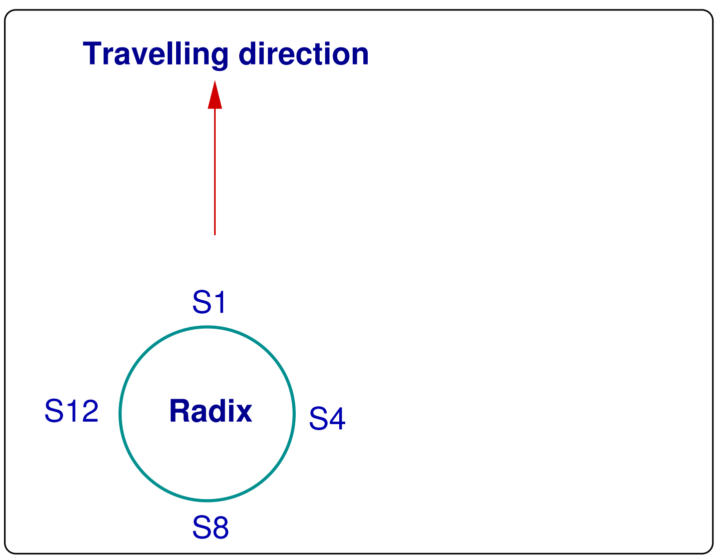
- NARMAX: non-linear ARMAX.
  See S. Chen and S.A. Billings, Representations of non-linear systems: The NARMAX model, Int. J. Control, 49, 1989, 1013-1032.

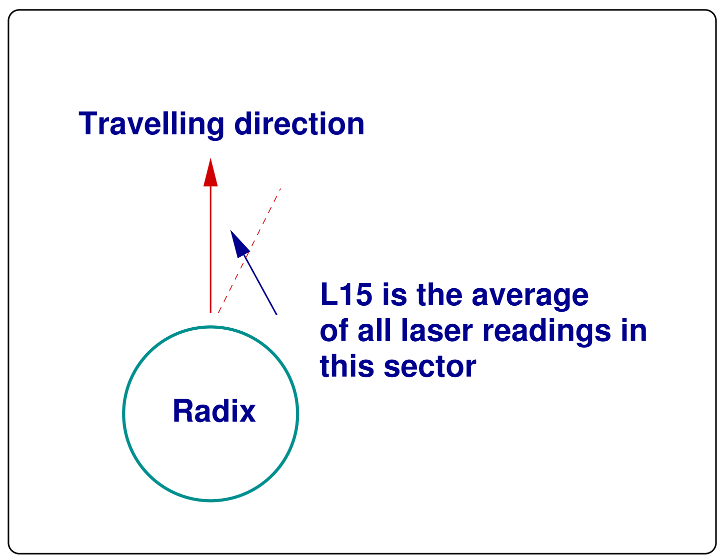
Advantages of the (N)ARMAX Modelling Approach

- Transparent
- Compact
- Analysable
- Transferable

Localisation: The ARMAX Identification Task







Model 1

	t	t-1	t-2	t-3	SSE
Laser 15	-47.40	-89.78	-72.11	-21.42	2506.4
Laser 30	-12.04	-14.14	-7.05	12.05	410.0
Laser 45	-17.19	-9.57	-10.05	-2.25	634.9
Laser 60	-3.37	4.99	-2.78	7.01	337.9
Laser 75	18.90	9.85	45.78	16.14	1784.3
Laser90	2.39	9.28	-7.01	-0.52	337.9
Laser 105	-14.71	-17.80	-9.74	-12.08	1435.6
Laser 120	23.16	19.02	18.91	-1.52	1734.7
Laser 135	-18.79	1.65	6.90	19.91	555.0
Laser 150	2.05	-6.87	-11.79	10.39	402.0
Laser 165	13.44	9.04	14.84	-2.08	1195.2
Laser 180	18.36	14.95	19.65	-5.86	1597.4
Sonar 1	-4.17	-3.14	-2.45	0.76	444.1
Sonar 2	-3.02	-1.93	-0.91	-1.19	474.0
Sonar 3	-3.27	1.52	2.98	2.15	364.0
Sonar 4	-0.92	-3.96	-4.57	-3.42	636.5
Sonar 5	-4.93	-5.51	-3.06	-1.11	710.4
Sonar 6	1.46	-0.80	-2.16	-3.92	418.0
Sonar 7	-1.02	-2.63	-4.06	-3.44	622.5
Sonar 8	0.99	0.99	0.99	0.99	428.8
Sonar 9	1.24	0.50	1.16	0.48	346.5
Sonar 10	1.15	2.72	0.59	-0.07	356.7
Sonar 11	-0.17	-0.39	-0.55	-0.41	317.5
Sonar 12	-0.39	0.63	0.46	1.39	322.5
Sonar 13	-0.49	0.94	2.83	-0.07	331.3
Sonar 14	1.97	2.20	2.78	1.79	374.3
Sonar 15	0.44	2.47	1.16	0.71	357.2
Sonar 16	-3.86	-1.90	-1.01	-0.88	422.4

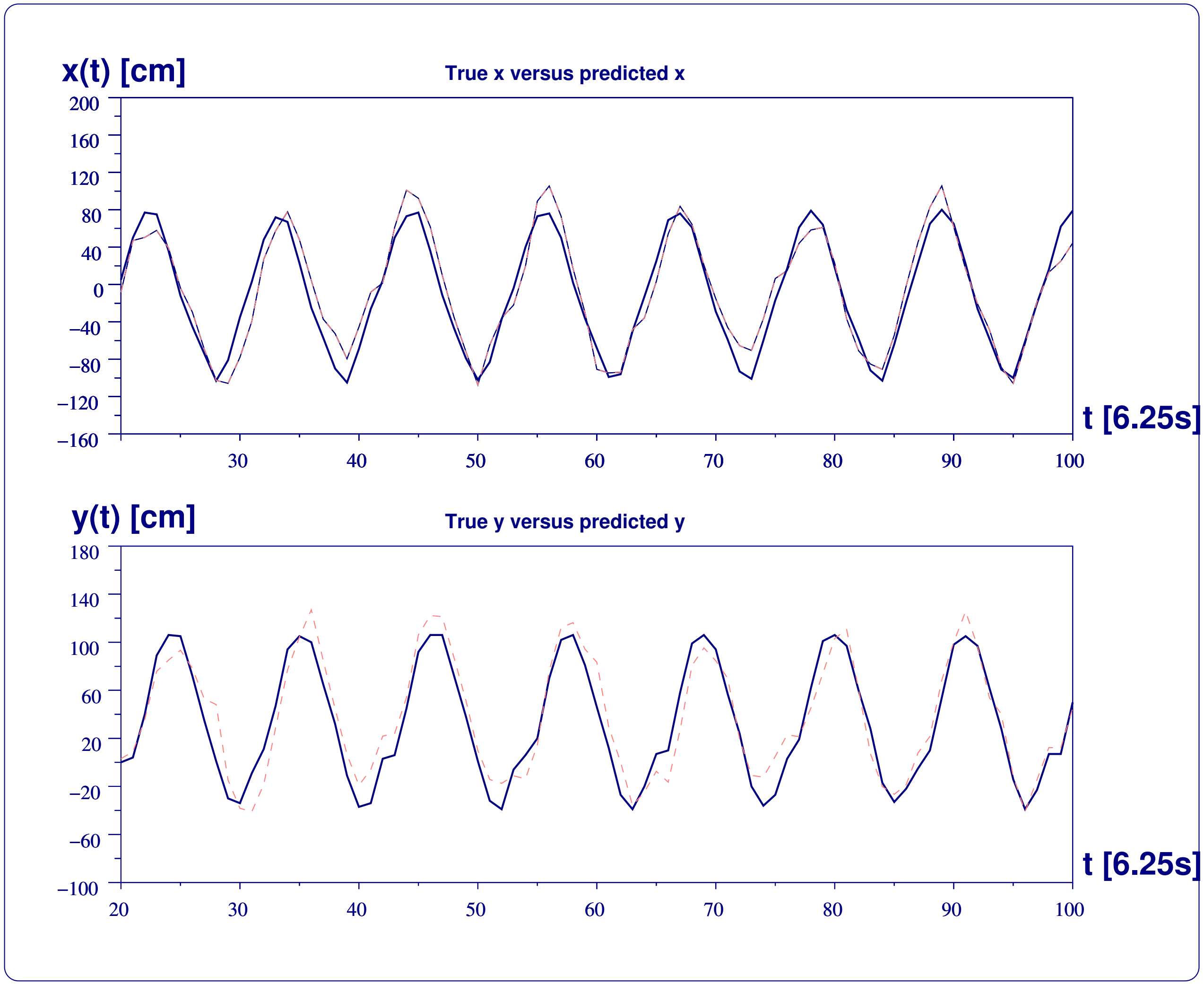
ARMAX model 1 of $x(t)$

SSE indicates the sum squared error if the respective term is removed from the model.

	t	t-1	t-2	t-3	t-4	SSE
Laser 15	35.65	7.86	-38.68	-74.11	-38.72	1223.4
Laser 30	8.65	-5.01	-7.89	-12.35	-3.82	378.9
Laser 45	8.18	-9.80	-0.75	-3.61	1.90	295.0
Laser 60	-5.96	-0.57	13.33	10.59	-6.89	317.6
Laser 75	3.74	5.79	0.27	29.41	26.38	1285.7
Laser 90	-11.34	1.64	1.91	-1.28	-1.10	358.7
Laser 105	3.52	-3.97	-14.41	-8.67	-5.30	817.6
Laser 120	7.41	17.19	20.37	16.40	13.45	2114.1
Laser 135	4.13	-7.20	-12.44	-9.44	9.32	623.9
Laser 150	0.58	10.29	-2.45	-8.90	1.23	320.3
Laser 165	-6.16	1.86	11.38	15.51	0.53	794.1
Laser 180	-10.22	2.12	4.88	18.30	9.67	852.5
Sonar 1	-1.64	-2.72	-1.97	-2.74	-1.15	455.6
Sonar 2	-0.42	-1.85	1.14	1.18	-0.66	280.5
Sonar 3	-1.98	-3.82	-0.68	0.99	0.72	353.7
Sonar 4	2.50	1.30	0.37	-3.46	-4.25	349.6
Sonar 5	0.16	-1.91	-2.42	-1.83	-1.61	446.0
Sonar 6	2.67	1.77	0.41	-0.79	-1.54	307.3
Sonar 7	2.12	1.56	-1.13	-2.55	-1.00	297.4
Sonar 8	-2.93	-2.93	-2.93	-2.93	-2.93	1069.0
Sonar 9	0.75	0.35	1.02	1.72	1.56	333.0
Sonar 10	-1.07	-0.35	2.37	1.30	1.37	301.0
Sonar 11	0.24	1.19	0.66	-0.86	0.73	282.2
Sonar 12	-0.06	0.48	0.61	0.61	1.72	285.4
Sonar 13	-1.29	0.14	1.48	3.80	2.89	318.9
Sonar 14	-1.95	0.19	1.61	2.51	2.01	292.1
Sonar 15	-0.77	0.15	0.96	1.11	0.58	284.3
Sonar 16	-1.21	-2.46	-1.59	-1.48	-0.09	392.6

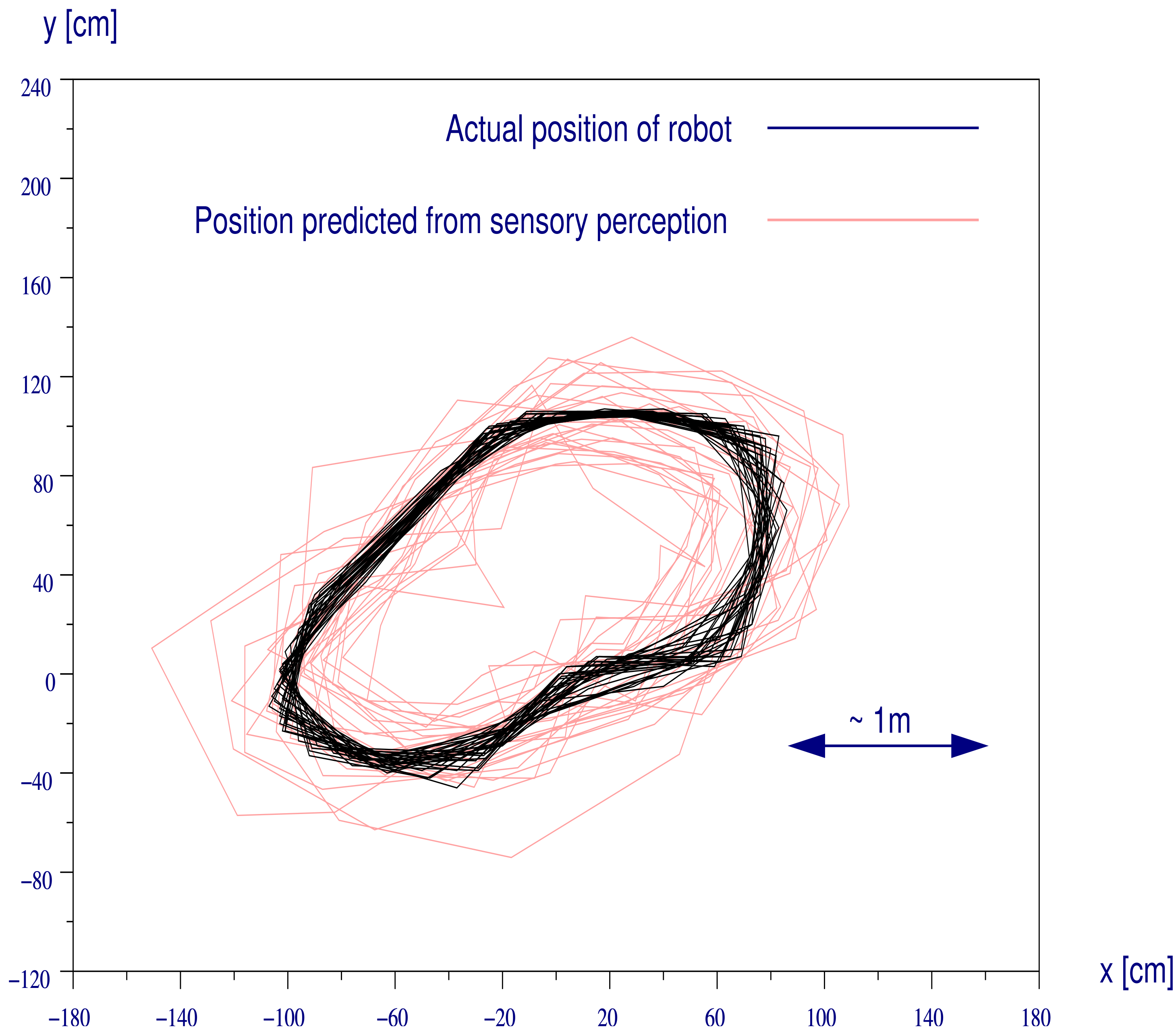
ARMAX model 1 of $y(t)$

Performance of Model 1

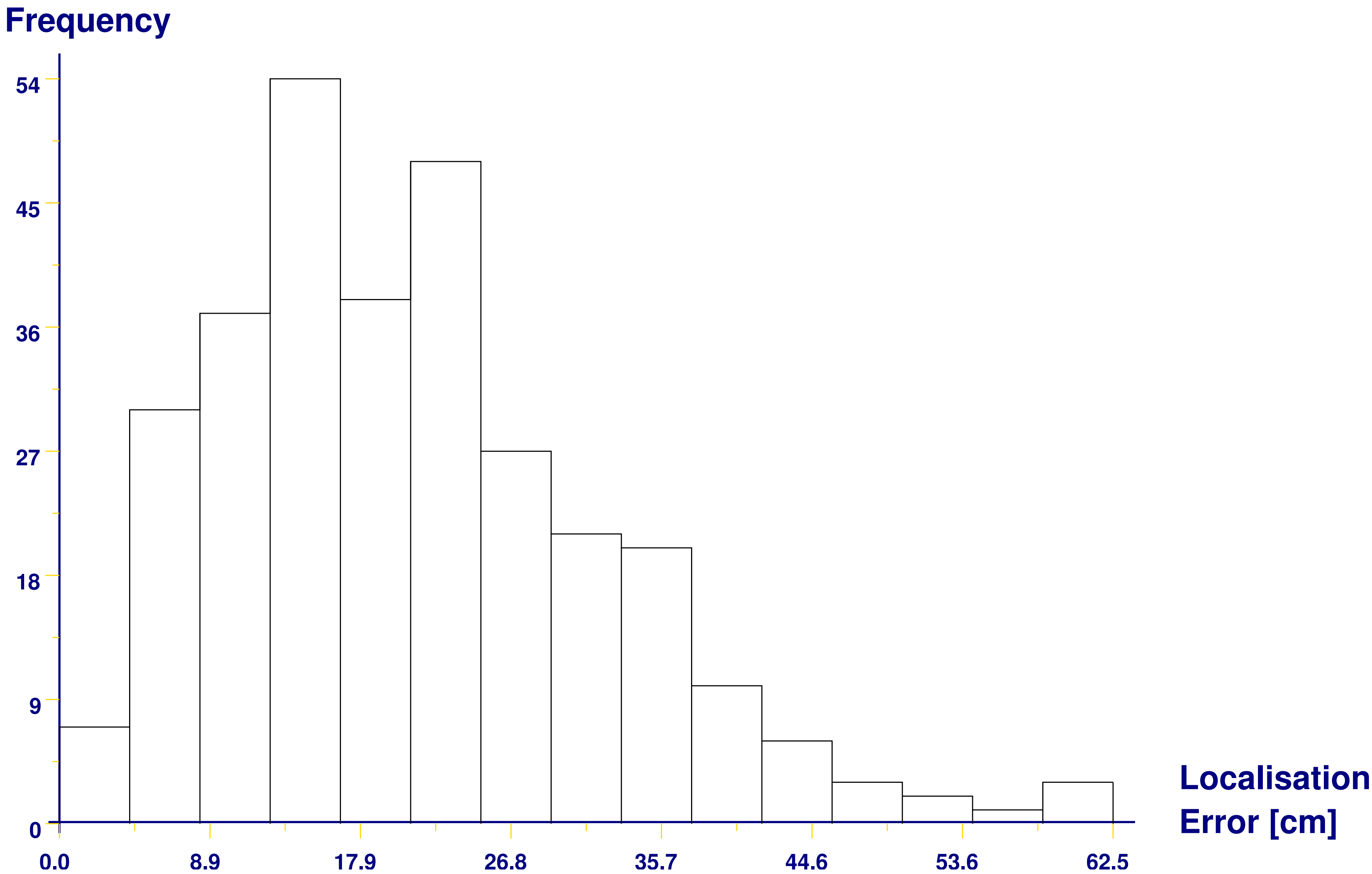


Actual robot position $< x(t), y(t) >$ (thick, bold line) versus the position $< \tilde{x}(t), \tilde{y}(t) >$ that is estimated from sensory perception, using model 1 (faint line)

$r_x = 0.96$, $r_y = 0.95$ (sig., p< 0.05)



Actual robot trajectory (thick, bold line) versus the trajectory that is estimated from sonar and laser perceptions, using model 1. Mean localisation error 29.5 cm $\pm$ 0.84 cm.



Distribution of localisation errors for the test data, using model 1

Refining Model 1

	t	t-1	t-2	t-3	SSE
Laser 15	-47.40	-89.78	-72.11	-21.42	2506.4
Laser 30	-12.04	-14.14	-7.05	12.05	410.0
Laser 45	-17.19	-9.57	-10.05	-2.25	634.9
Laser 60	-3.37	4.99	-2.78	7.01	337.9
Laser 75	18.90	9.85	45.78	16.14	1784.3
Laser90	2.39	9.28	-7.01	-0.52	337.9
Laser 105	-14.71	-17.80	-9.74	-12.08	1435.6
Laser 120	23.16	19.02	18.91	-1.52	1734.7
Laser 135	-18.79	1.65	6.90	19.91	555.0
Laser 150	2.05	-6.87	-11.79	10.39	402.0
Laser 165	13.44	9.04	14.84	-2.08	1195.2
Laser 180	18.36	14.95	19.65	-5.86	1597.4
Sonar 1	-4.17	-3.14	-2.45	0.76	444.1
Sonar 2	-3.02	-1.93	-0.91	-1.19	474.0
Sonar 3	-3.27	1.52	2.98	2.15	364.0
Sonar 4	-0.92	-3.96	-4.57	-3.42	636.5
Sonar 5	-4.93	-5.51	-3.06	-1.11	710.4
Sonar 6	1.46	-0.80	-2.16	-3.92	418.0
Sonar 7	-1.02	-2.63	-4.06	-3.44	622.5
Sonar 8	0.99	0.99	0.99	0.99	428.8
Sonar 9	1.24	0.50	1.16	0.48	346.5
Sonar 10	1.15	2.72	0.59	-0.07	356.7
Sonar 11	-0.17	-0.39	-0.55	-0.41	317.5
Sonar 12	-0.39	0.63	0.46	1.39	322.5
Sonar 13	-0.49	0.94	2.83	-0.07	331.3
Sonar 14	1.97	2.20	2.78	1.79	374.3
Sonar 15	0.44	2.47	1.16	0.71	357.2
Sonar 16	-3.86	-1.90	-1.01	-0.88	422.4

ARMAX model 1 of $x(t)$

Model 2: Refinement of Model 1

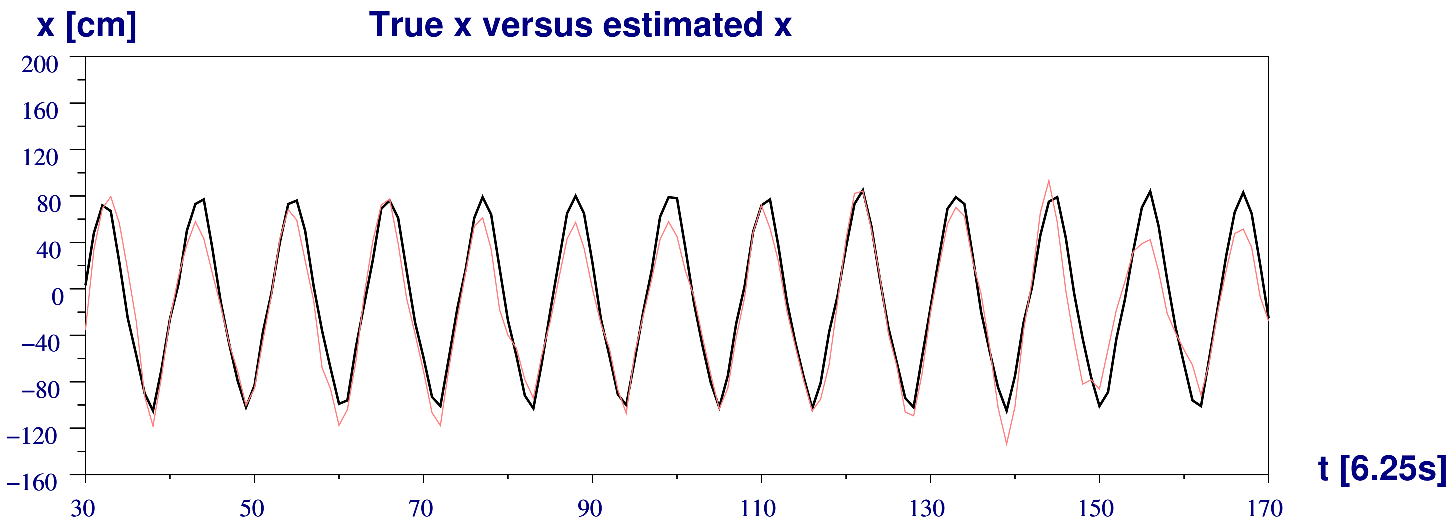
	t-0	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11	t-12	SSE
L15	-56	-73	-76	-54	-5	45	67	68	51	27	-13	-39	-58	1541
L75	-	3	8	4	-2	-6	-13	-13	-13	-13	4	13	17	388
L105	-13	-12	-9	-5	7	6	9	8	-0.2	-5	-14	-14	-8	1388
L120	18	13	7	-7	-13	-19	-17	-6	6	11	9	6	6	511
L165	12	16	14	9	5	0.7	-10	-7	-7	2	13	9	9	2133
L180	-4	-1	-3	-2	0.1	4	-1	-6	-4	-0.2	1	8	12	354

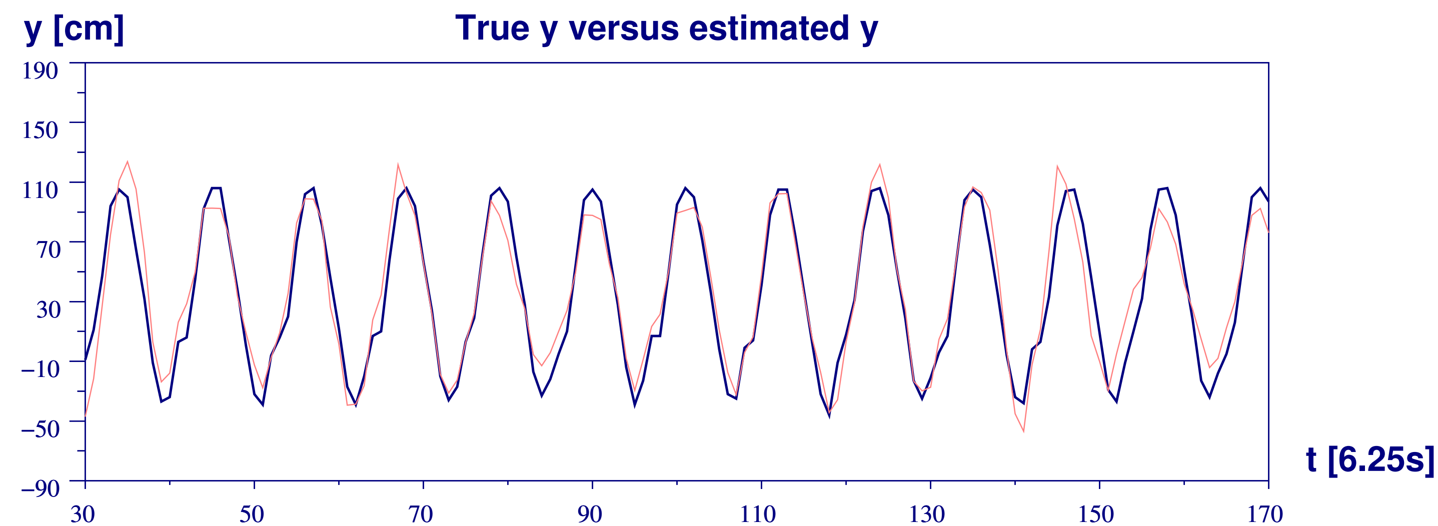
Alternative model (model 2) to determine x from laser sensor signals

t-0	t-1	t-2	t-3	t-4	t-5	t-6	t-7	t-8	t-9	t-10	t-11	t-12	t-13	SSE
12	-14	-40	-71	-64	-46	8	38	48	62	49	13	-11	-41	961
0.5	0	-0.1	13	8	4	3	-9	-10	-12	-16	-5	6	10	274
7	-5	-10	-8	-8	-2	-0.2	8	9	8	3	-9	-11	-11	784.
6	19	15	10	5	-1	-16	-15	-12	-6	4	8	7	6	926
-8	4	11	15	11	7	5	-6	-7	-11	-4	7	7	8	1245
-4	-1	-2	-1	-1	0	1	-0.2	-2	-3	0.2	-2	4	11	263

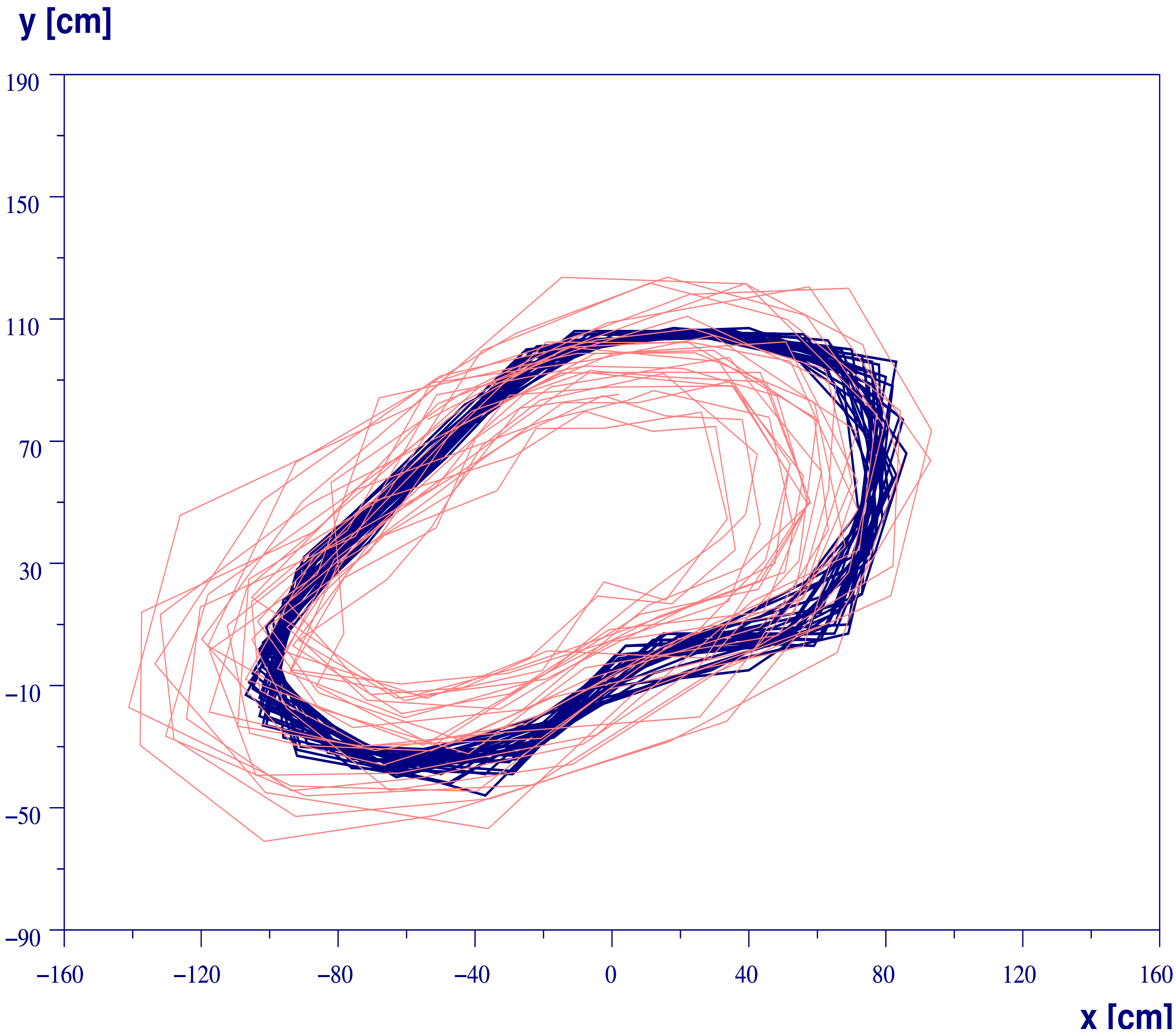
Alternative model (model 2) to determine y from laser sensor signals

Performance of Model 2

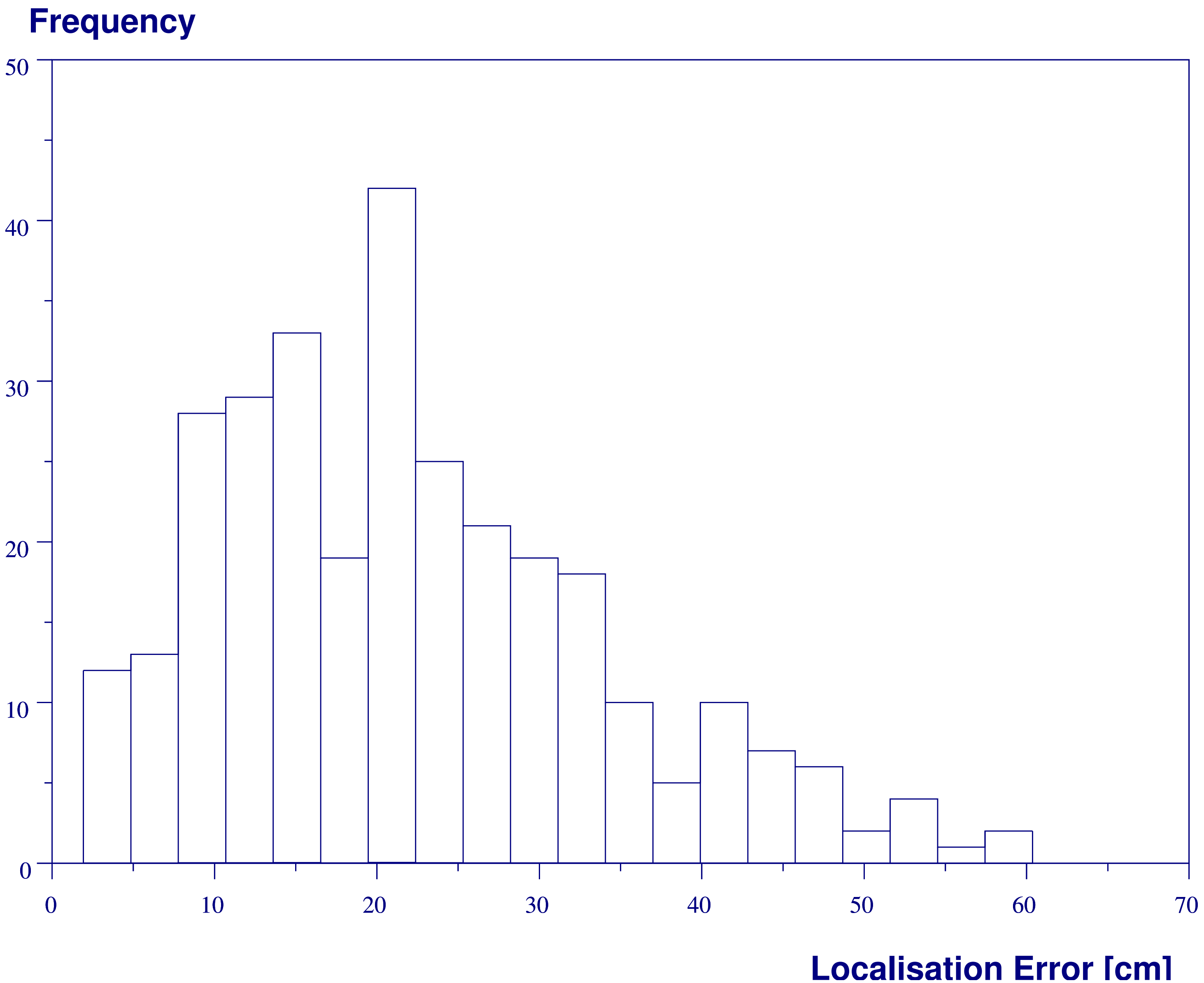




Actual robot position $< x(t), y(t) >$ (thick, bold line) versus the position that is estimated from sensory perception, using the alternative model 2 (faint line)



Actual robot trajectory (thick, bold line) versus the trajectory that is estimated from laser perception, using model 2 (faint line). Mean localisation error 22 cm ± 0.7 cm.



Distribution of localisation errors for the test data, using model 2

Model 3: Self-Localisation Using Perception and Action

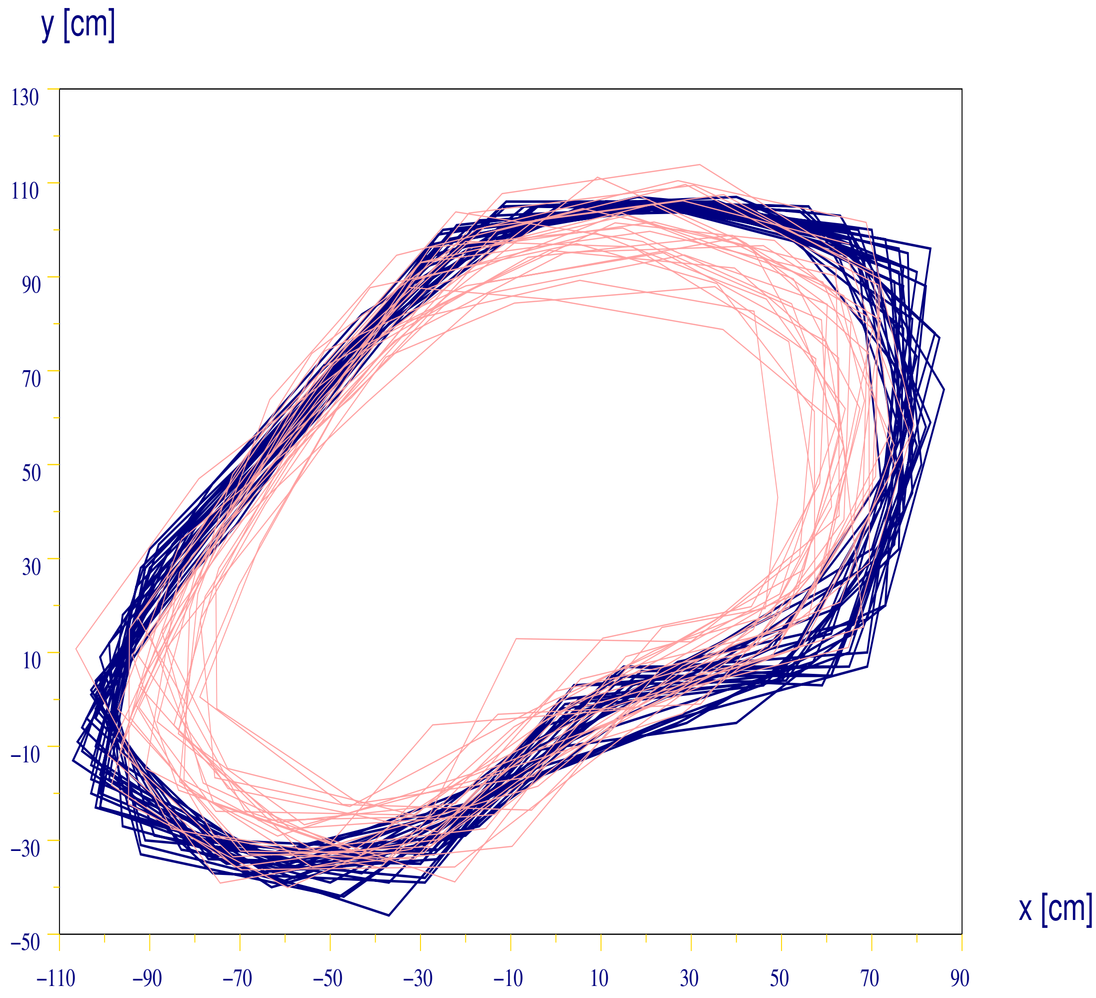
	t	t-1	t-2	t-3	t-4
x		1.06	- 0.72	0.65	- 0.70
L15	- 28.92	- 7.01	- 0.54	- 6.80	
L75	- 2.99	+ 7.08	- 2.78	+ 10.33	
L105	3.91	- 1.50	- 3.49	+ 1.68	
L120	3.63	+ 1.28	- 3.85	- 0.24	
L135	- 7.40	+ 2.14	+ 6.34	+ 0.01	
L180	2.59	+ 3.32	- 8.06	+ 3.42	

Model 3 for $x(t)$, taking previous x estimates into account

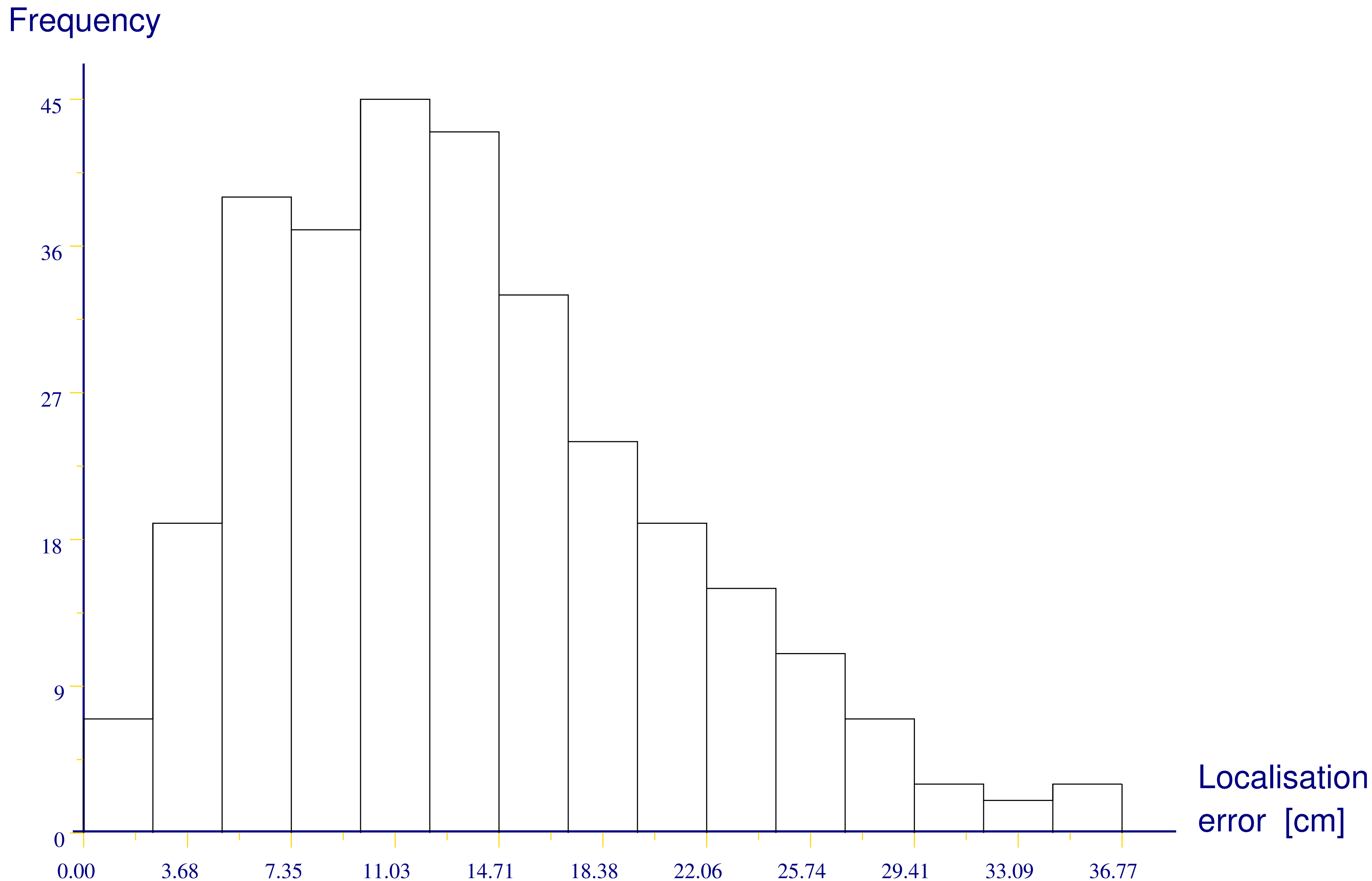
	t	t-1	t-2	t-3	t-4
y		0.85	- 0.18	0.09	- 0.49
L15	9.96	- 5.83	- 15.84	- 8.31	+ 0.88
L30	16.80	- 9.71	- 5.20	+ 2.48	+ 0.34
L75	1.46	+ 0.41	- 5.68	+ 7.11	- 0.48
L120	1.95	+ 2.97	+ 4.06	- 4.00	+ 7.88
L135	1.99	+ 1.03	- 5.28	+ 2.58	+ 5.50
L180	- 2.62	+ 1.80	+ 4.10	- 3.82	+ 0.34

Model 3 for $y(t)$, taking previous y estimates into account

Performance of Model 3



Actual robot trajectory (thick, bold line) versus the trajectory that is estimated from sonar perception, using model 3 (faint line).
Mean localisation error 13 cm $\pm$ 0.4 cm.
$r_x = r_y = 0.98$ (sig., $p < 0.05$)



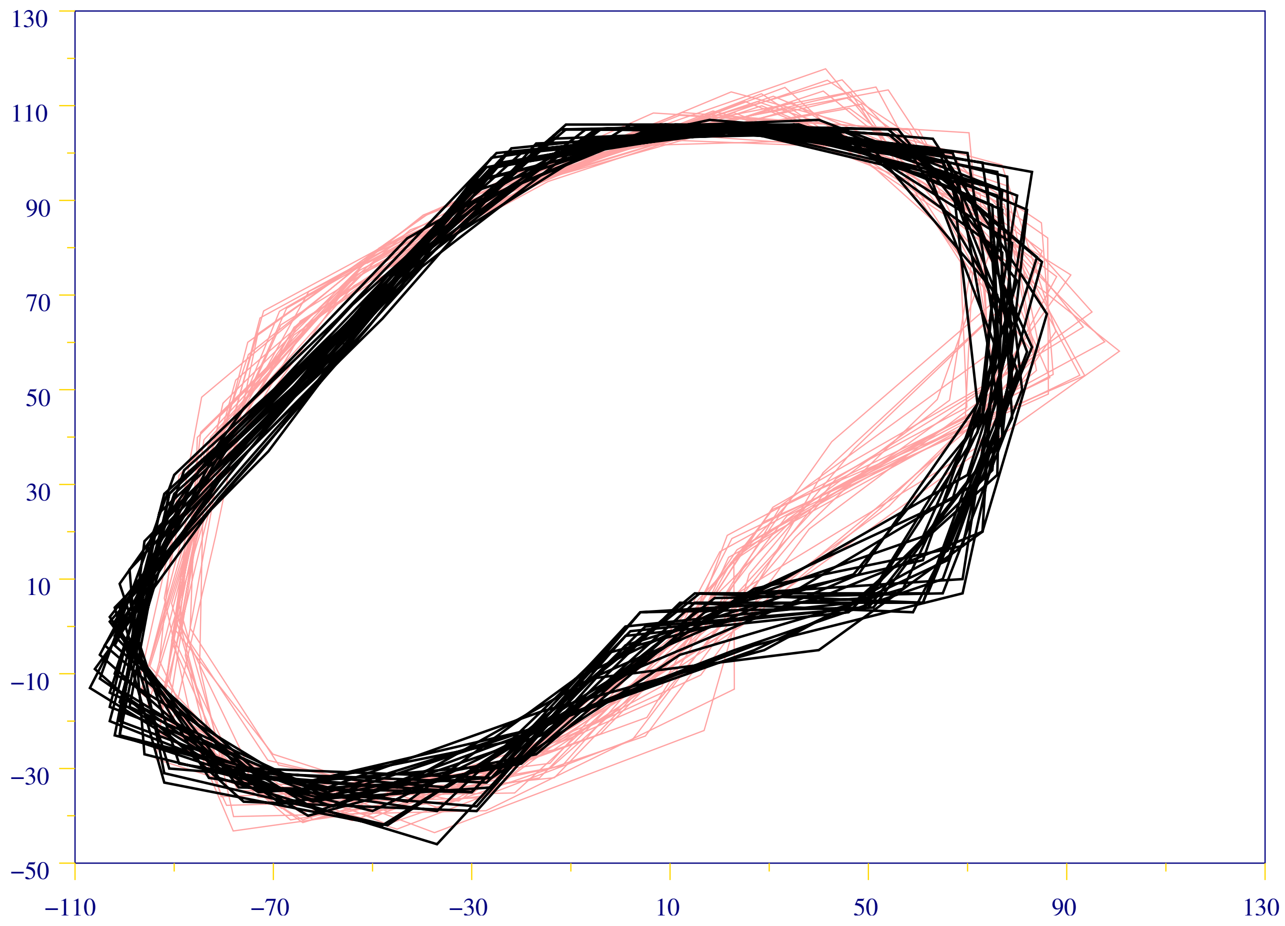
Distribution of localisation errors for the test data, using model 3

Model 4: Refinement of Model 3

$$
\begin{aligned}
x(t) = {} & 1.04x(t-1) - 0.65y(t-1) \\
& -6.7L_{15}(t) - 8.84L_{30}(t) + 4.05_{L75}(t) \\
& +7.1L_{120}(t) - 5.75L_{135}(t) + 14.53L_{180}(t)
\end{aligned}
\tag{1}
$$

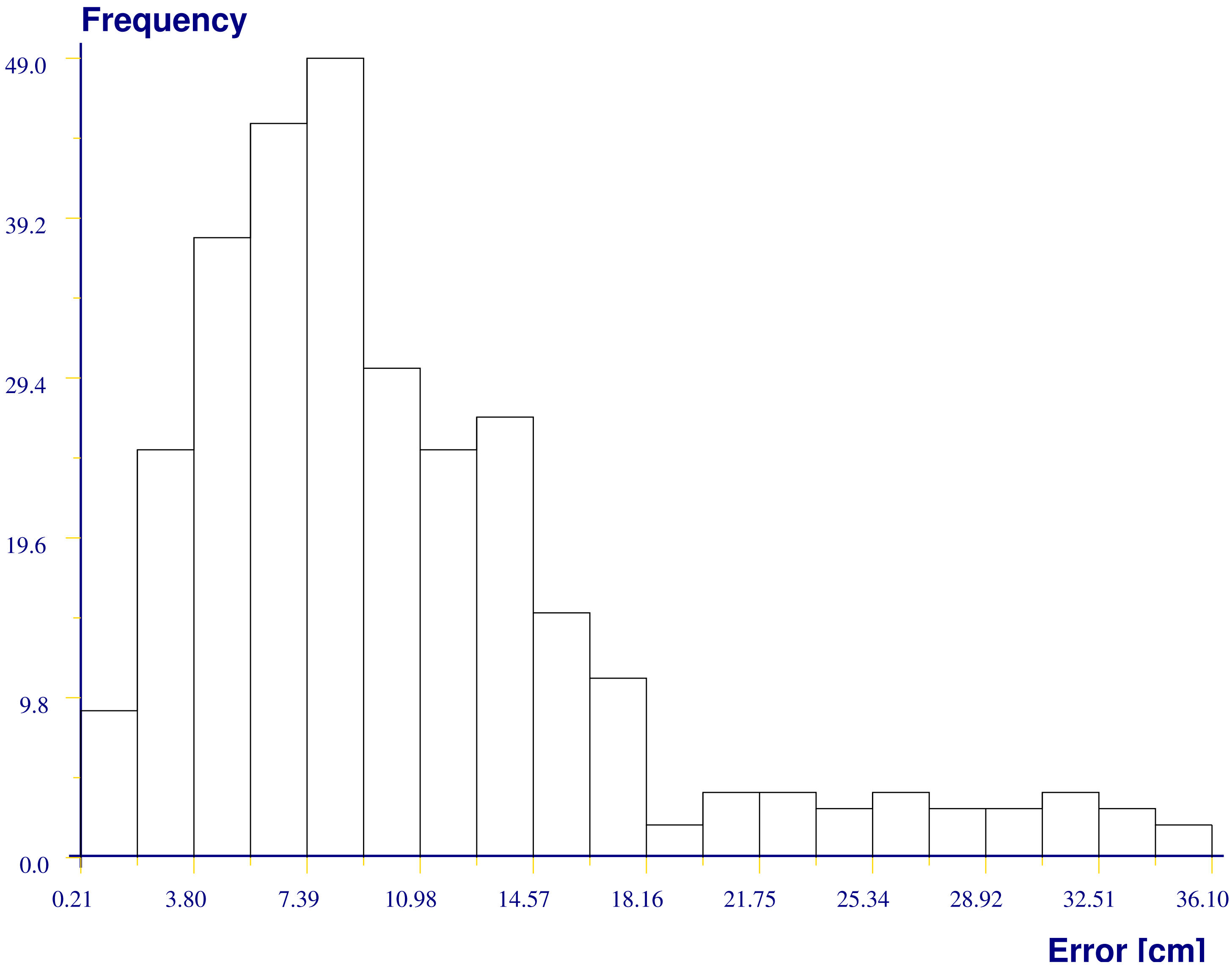
$$
\begin{aligned}
y(t) = {} & -0.49x(t-1) - 0.56y(t-1) \\
& -2.54L_{15}(t) + 11.43L_{30}(t) + 2.19L_{75}(t) \\
& +5.93L_{120}(t) - 2.14L_{135}(t) + 3.10L_{180}(t)
\end{aligned}
\tag{2}
$$

Performance of Model 4



Actual robot trajectory (thick, bold line) versus the trajectory that is estimated from previous position $< x, y >$ and laser perception, using model 4.

Mean localisation error 10.5 cm $\pm$ 0.4 cm.

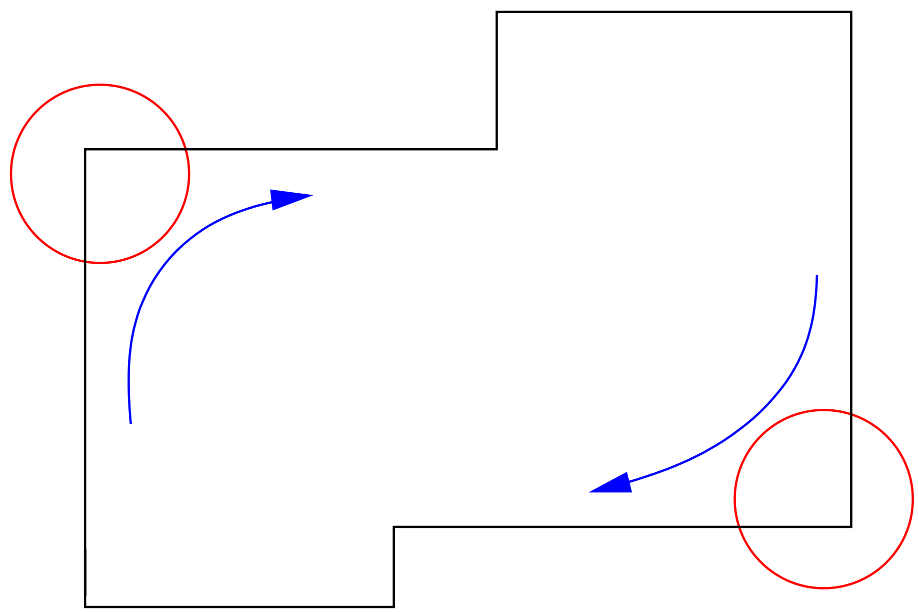


Distribution of localisation errors for the test data, using model 4

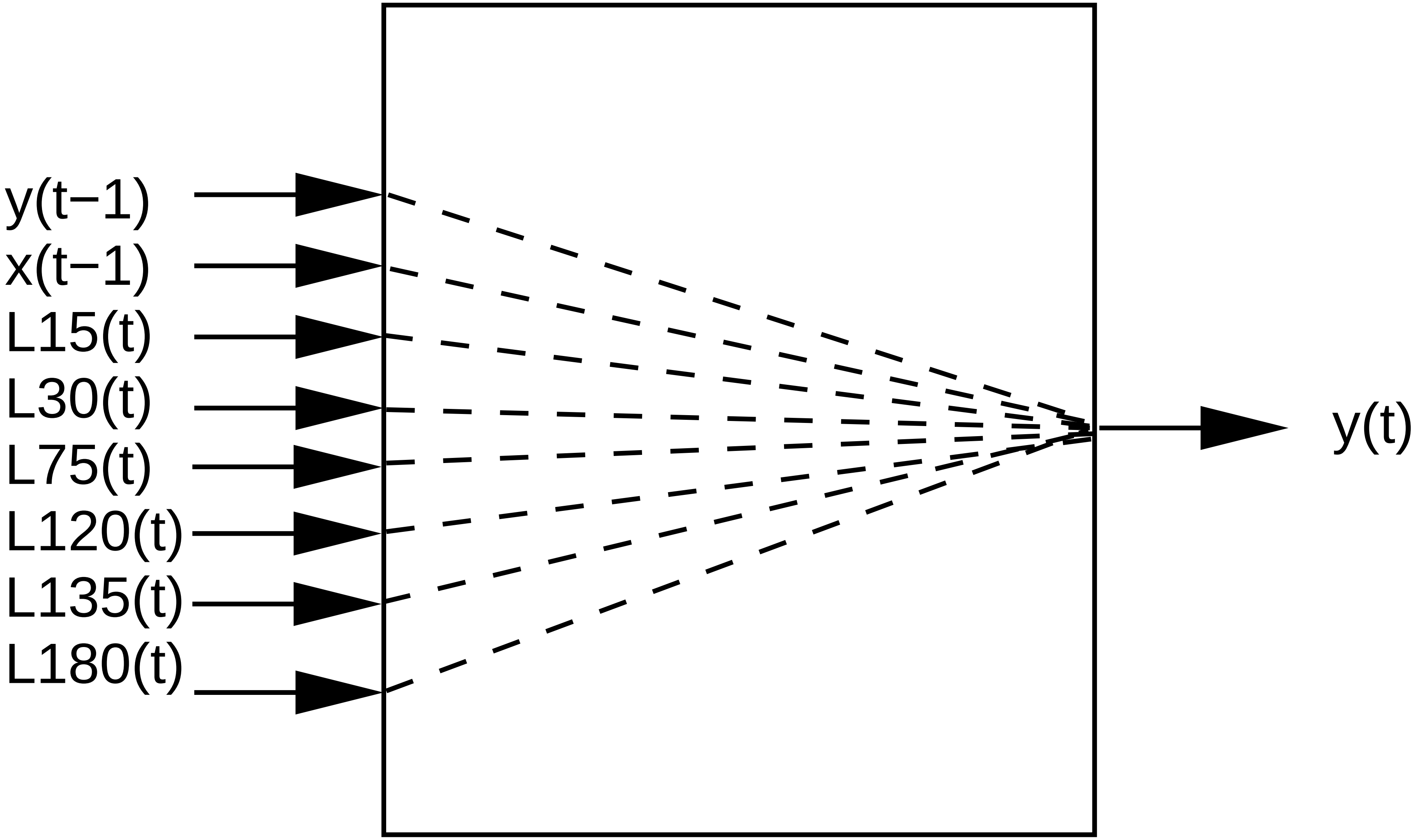
What does a Model mean?

$$
\begin{aligned}
x(t) = {} & 1.04x(t-1) - 0.65y(t-1) \\
& -6.7L_{15}(t) - 8.84L_{30}(t) + 4.05_{L75}(t) \\
& +7.1L_{120}(t) - 5.75L_{135}(t) + 14.53L_{180}(t)
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
y(t) = {} & -0.49x(t-1) - 0.56y(t-1) \\
& -2.54L_{15}(t) + 11.43L_{30}(t) + 2.19L_{75}(t) \\
& +5.93L_{120}(t) - 2.14L_{135}(t) + 3.10L_{180}(t)
\end{aligned} \tag{4}
$$

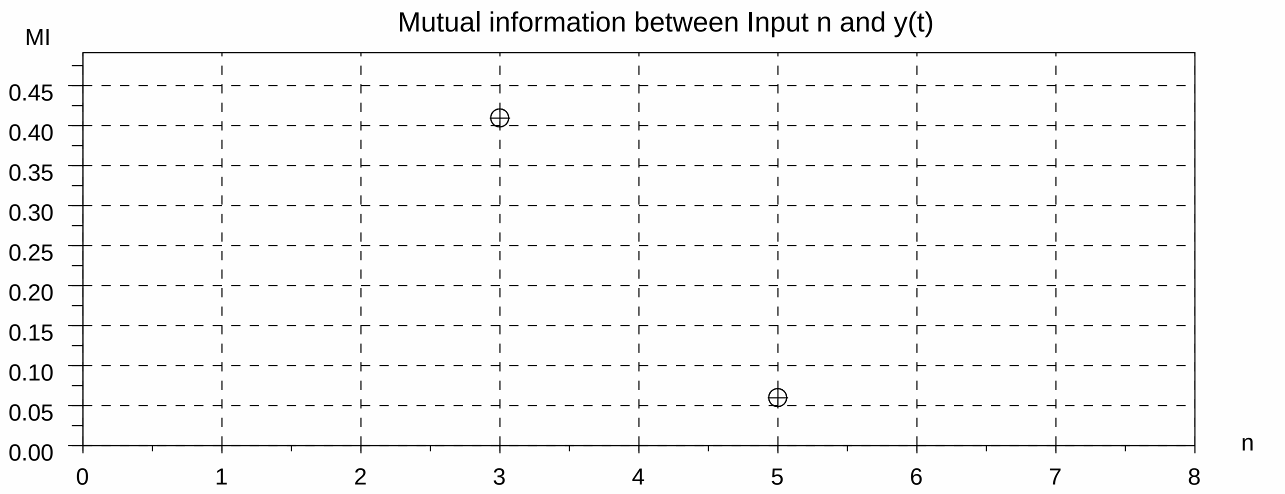


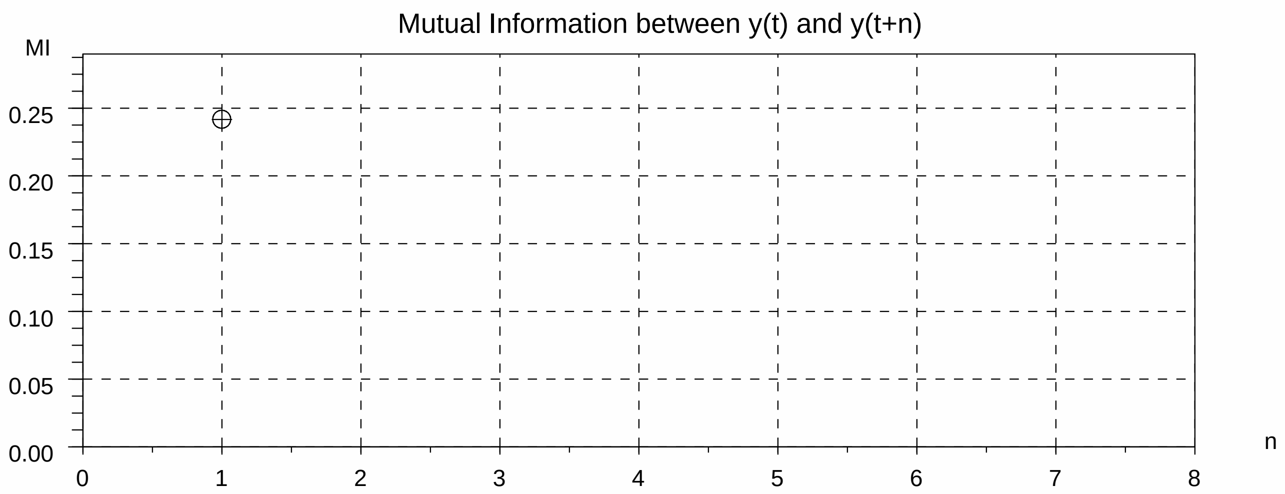
Sensitivity Estimate Using Mutual Information



Analysis of Model 4

$$y(t) = -0.49x(t-1) - 0.56y(t-1)$$
$$-2.54L_{15}(t) + 11.43L_{30}(t) + 2.19L_{75}(t)$$
$$+5.93L_{120}(t) - 2.14L_{135}(t) + 3.10L_{180}(t)$$

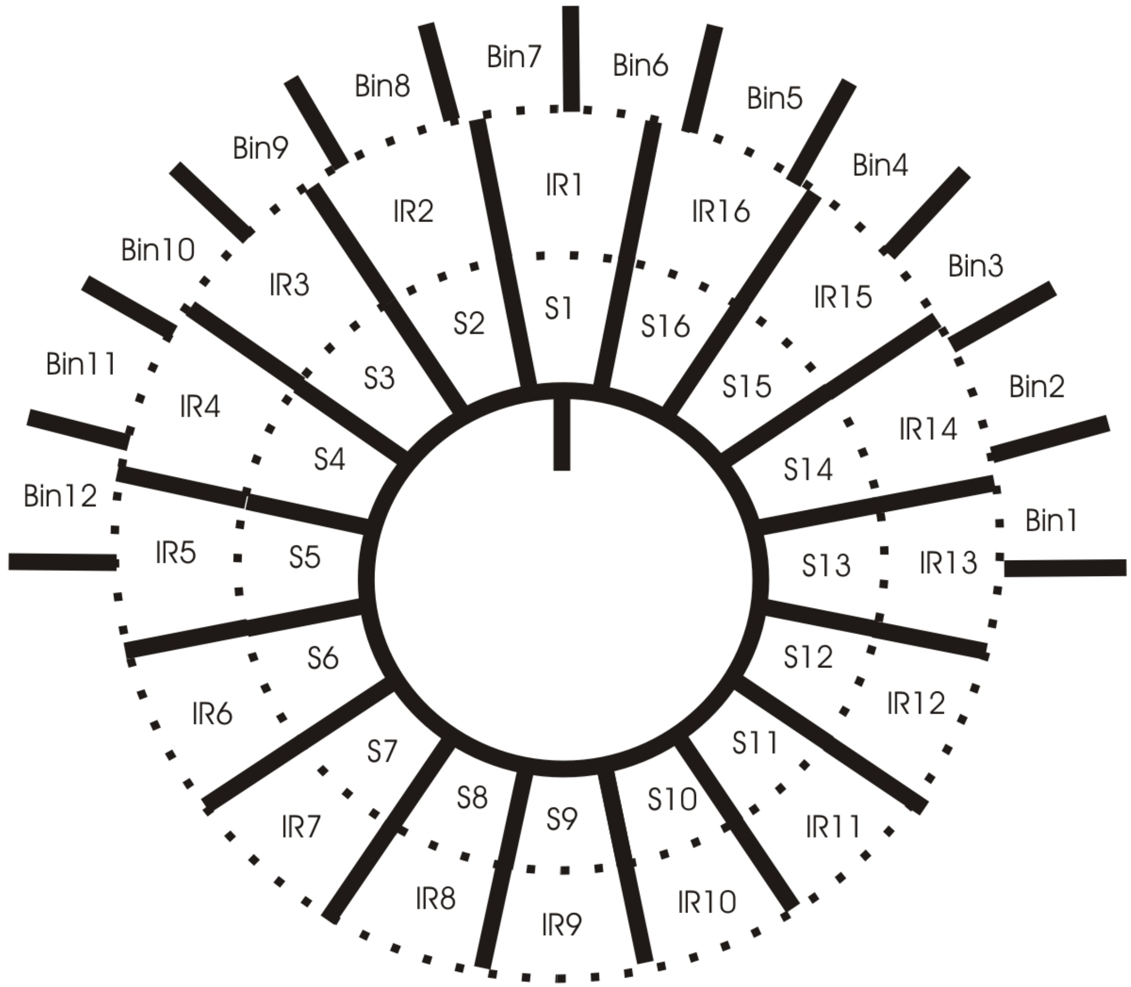




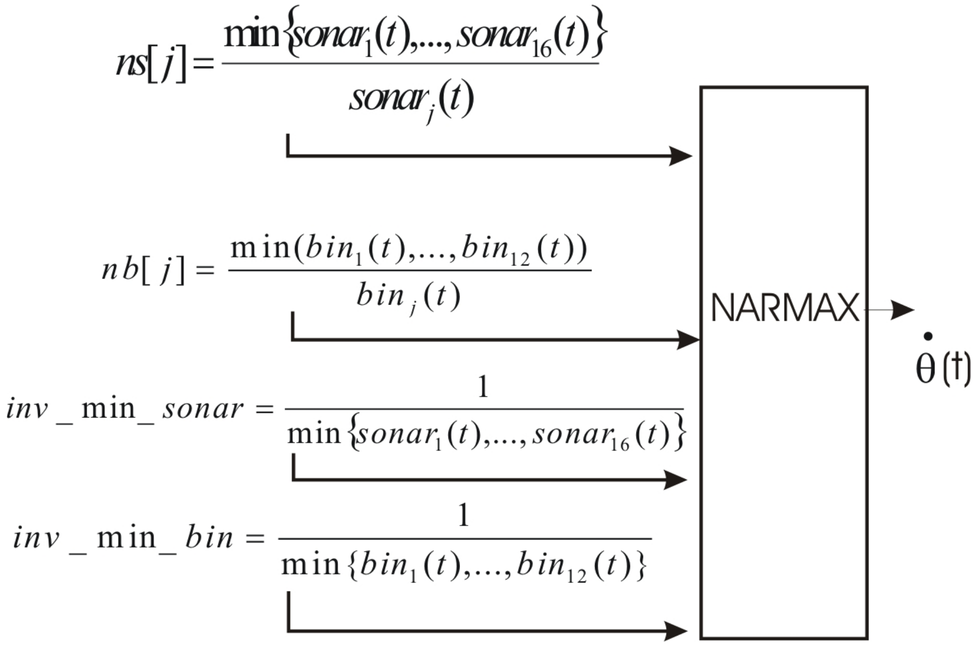
Experiment 2: Route Learning

Encoding of Sensory Perception

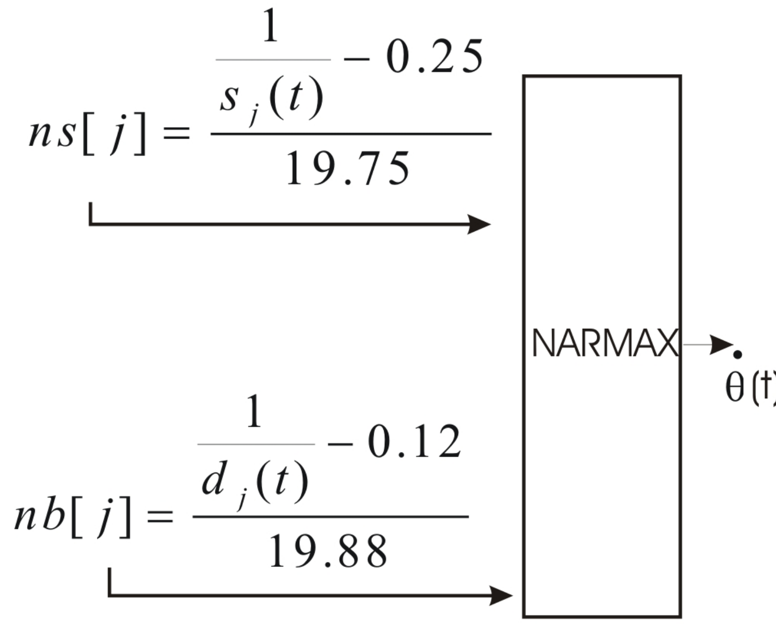
A)



B)

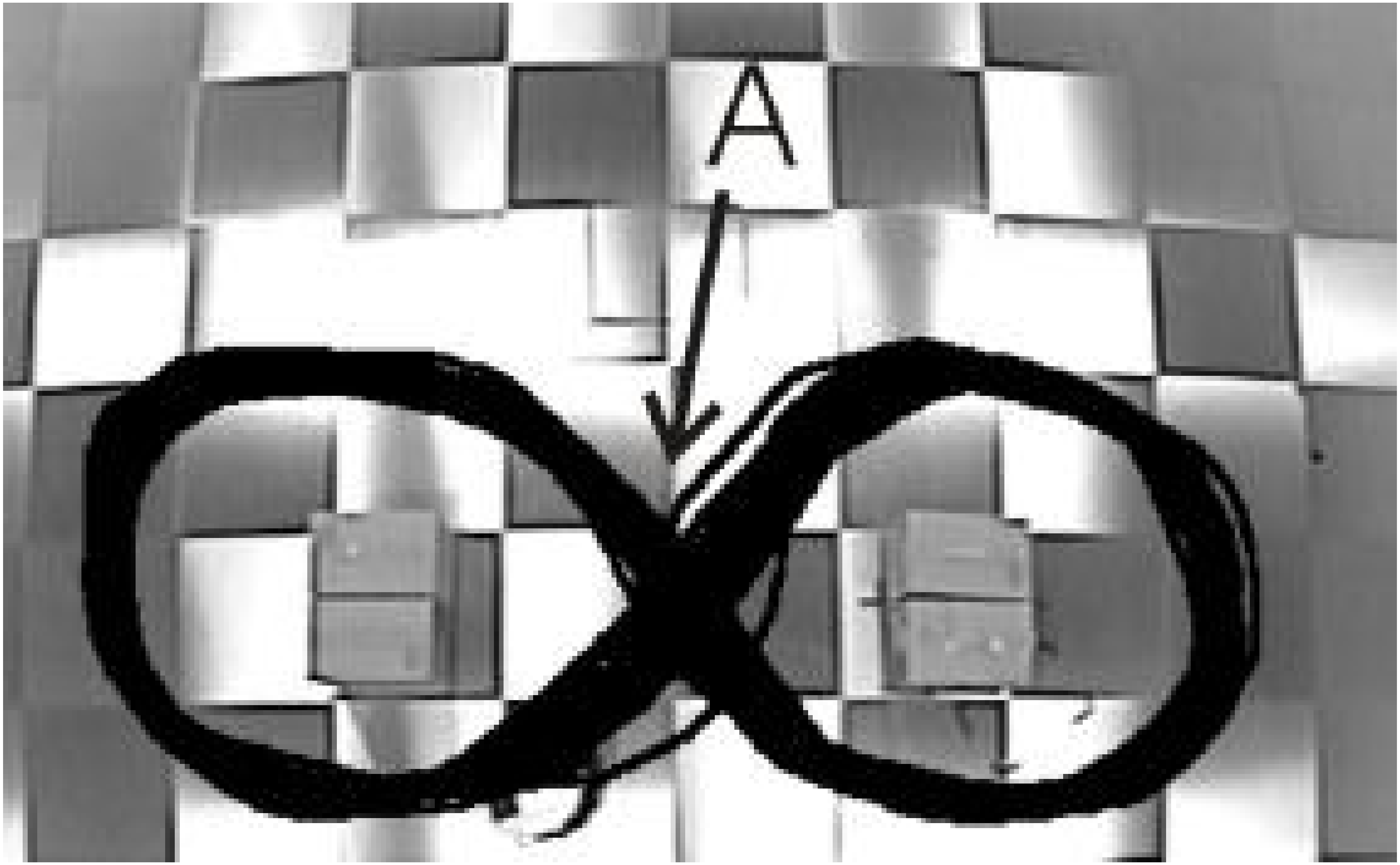

$ns[j] = \frac{\min\{sonar_1(t),...,sonar_{16}(t)\}}{sonar_j(t)}$
$nb[j] = \frac{\min(bin_1(t),...,bin_{12}(t))}{bin_j(t)}$
$inv\_min\_sonar = \frac{1}{\min\{sonar_1(t),...,sonar_{16}(t)\}}$
$inv\_min\_bin = \frac{1}{\min\{bin_1(t),...,bin_{12}(t)\}}$
NARMAX
$\dot{\theta}(t)$

A)


$ns[j] = \frac{\frac{1}{s_j(t)} - 0.25}{19.75}$
$nb[j] = \frac{\frac{1}{d_j(t)} - 0.12}{19.88}$
NARMAX
$\dot{\theta}(t)$

B)

Route Learning: Results

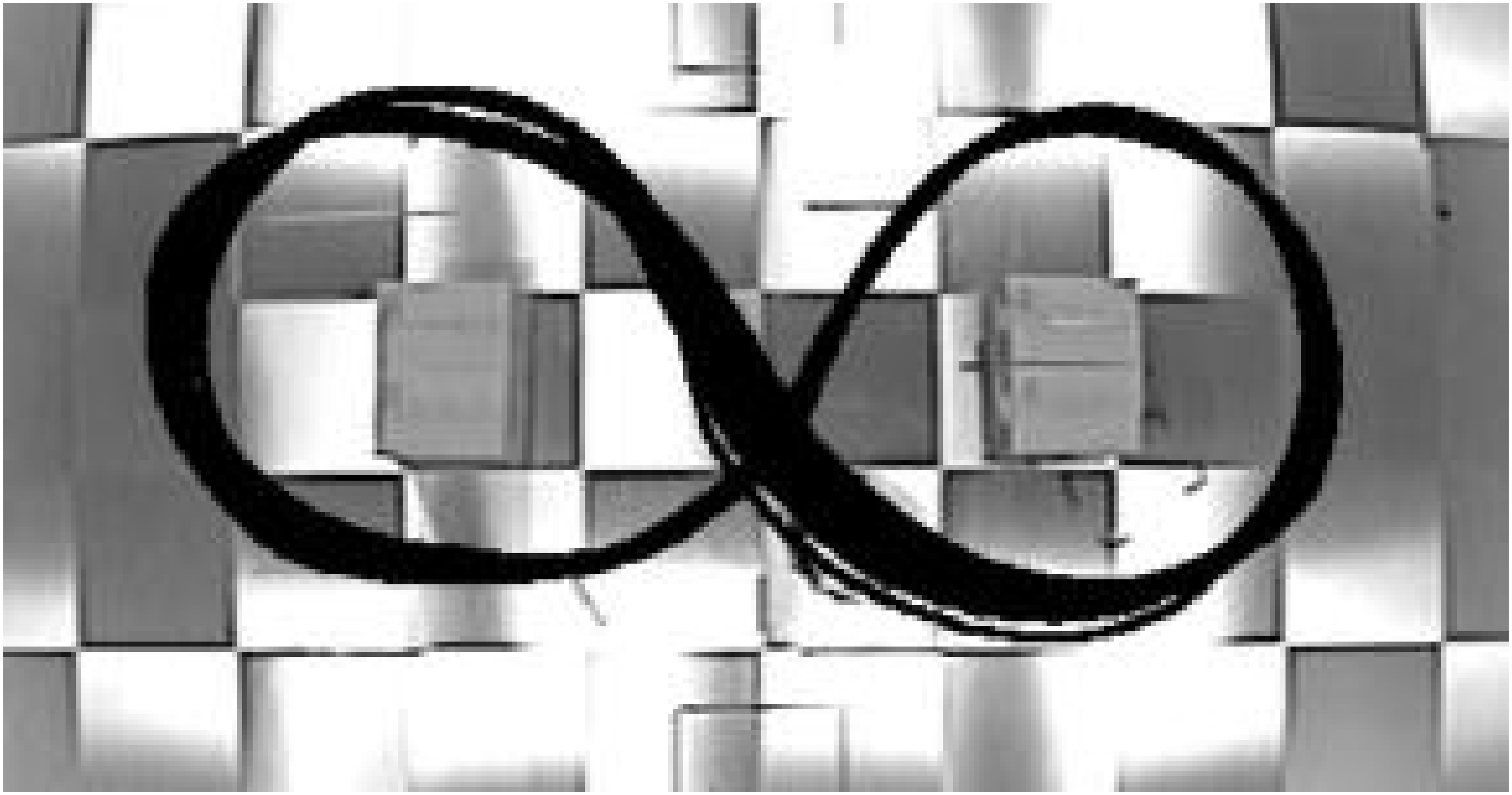


Route 1: Training route steered by a human operator

$$
\begin{aligned}
&\dot{\theta}(t) = +0.08 - 0.50 * nb[1] - 0.62 * nb[4] + 0.46 * nb[6] + 0.07 * nb[7] - \\
&+0.14 * nb[9] + 0.02 * nb[10] + 0.20 * nb[12] - 0.88 * ns[1] + 0.22 * ns[3] \\
&+0.004 * ns[6] - 0.04 * ns[7] + 0.20 * ns[10] - 0.02 * ns[14] + 0.11 * ns[1 \\
&-0.43 * inv\_min\_bin + 0.041 * nb[1]^2 + 0.02 * nb[2]^2 - 0.06 * nb[3]^2 + 0.5 \\
&-0.44 * nb[6]^2 + 0.01 * nb[9]^2 - 8.70 * inv\_min\_bin^2 - 0.07 * nb[1] * nb[2] \\
&+0.07 * nb[1] * nb[3] + 0.44 * nb[1] * nb[4] + 0.40 * nb[1] * nb[10] - 0.24 * \\
&+0.83 * nb[1] * ns[1] + 0.09 * nb[1] * ns[4] - 0.79 * nb[1] * ns[11] - 0.04 * \\
&+0.08 * nb[1] * inv\_min\_sonar + 3.58 * nb[1] * inv\_min\_bin + 0.36 * nb[2] \\
&-0.73 * nb[2] * nb[9] - 0.05 * nb[2] * nb[12] + 0.04 * nb[2] * ns[13] + 0.63 \\
&-0.28 * nb[3] * nb[12] + 0.11 * nb[3] * ns[3] - 0.48 * nb[4] * nb[10] - 0.27 * \\
&+0.11 * nb[5] * ns[1] + 0.26 * nb[6] * nb[8] + 0.02 * nb[7] * ns[5] + 0.15 * \\
&-0.18 * nb[7] * ns[12] - 0.17 * nb[8] * nb[10] + 0.03 * nb[8] * ns[5] - 0.10 * \\
&+0.05 * nb[10] * ns[12] + 0.03 * nb[12] * ns[10] + 0.06 * nb[12] * ns[11] + \\
&+0.01 * nb[12] * ns[16] + 2.68 * ns[1] * ns[6] - 0.30 * ns[1] * ns[11] - 1.9 \\
&+3.91 * ns[2] * inv\_min\_bin + 0.13 * ns[3] * ns[5] - 1.27 * ns[3] * ns[6] - \\
&-1.85 * ns[3] * inv\_min\_bin + 0.05 * ns[4] * ns[11] - 0.13 * ns[4] * inv\_mi \\
&-0.23 * ns[6] * ns[10] + 0.89 * ns[6] * ns[11] + 0.08 * ns[15] * ns[16] + \\
&+5.06 * ns[16] * inv\_min\_sonar
\end{aligned}
$$

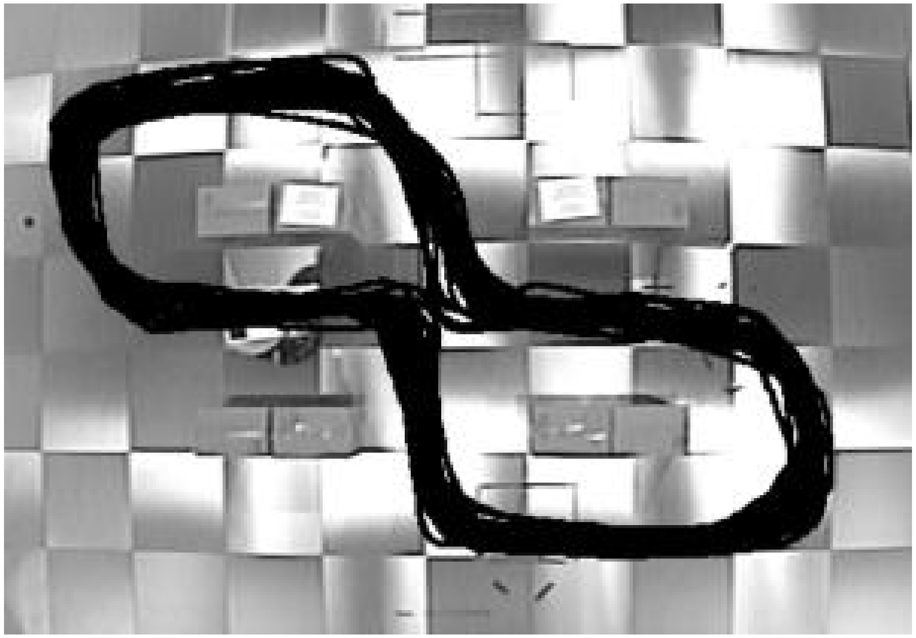
NARMAX model of the angular velocity $\dot{\theta}$ for the route following behaviour in route 1.

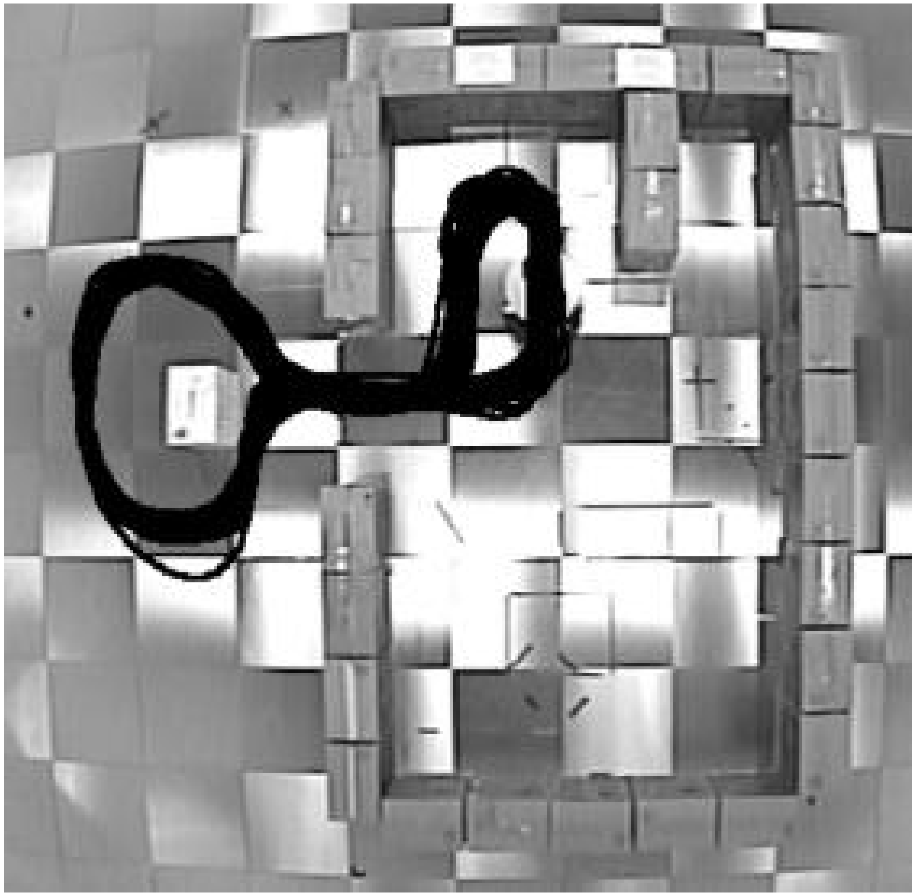
nb, ns, inv_min_sonar, and inv_min_laser, are the normalised bins, normalised sonar, inverted minimum sonar and laser readings respectively.

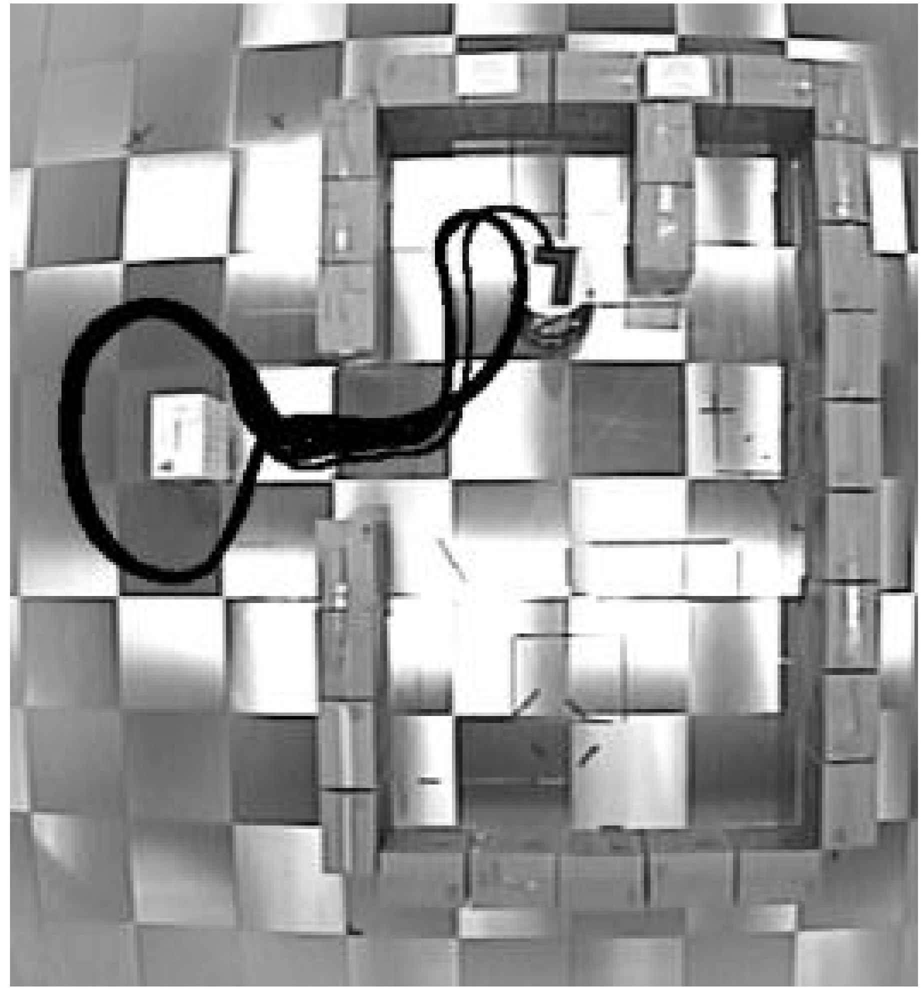


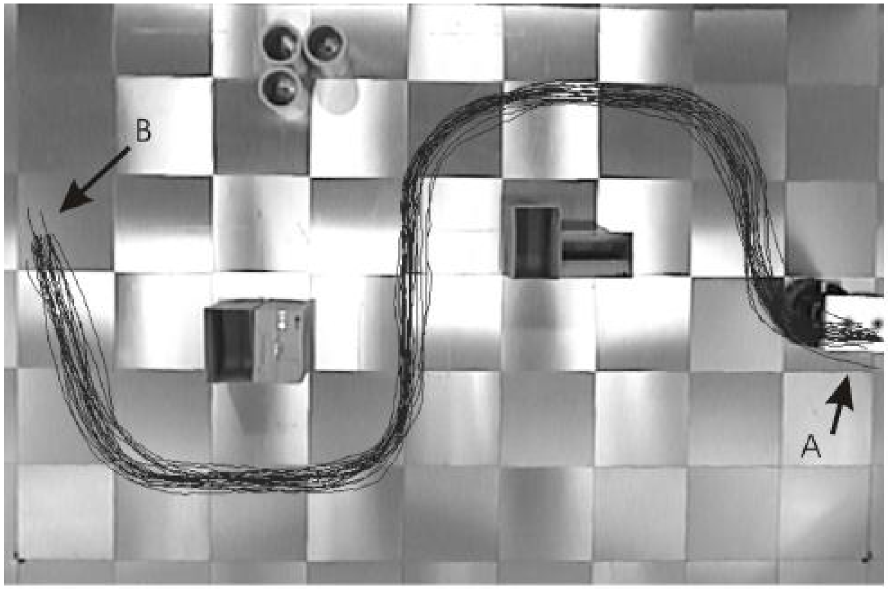
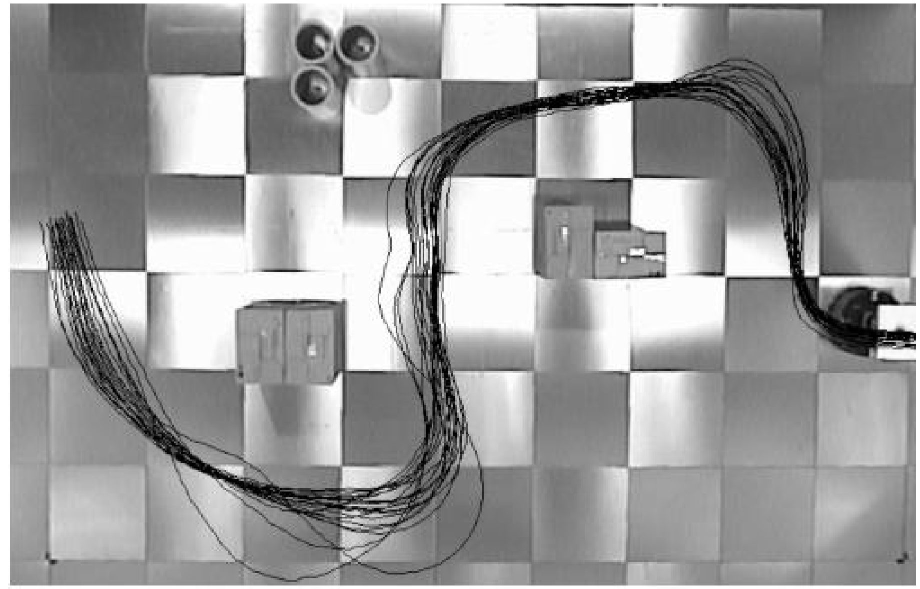
Route taken by the robot, operating autonomously under model control.

Other Routes








B
A


RobotMODIC: Relevance

1. Canonical representation of behaviour-describing aspects

   - Simple transfer for competencies between platform, research groups, *etc*
   - Implementation is very efficient, i.e. fast in execution with low memory requirements
   - Closed mathematical form aids visualisation, modification and analysis

RobotMODIC: Relevance

1. Canonical representation of behaviour-describing aspects
   - Simple transfer for competencies between platform, research groups, *etc*
   - Implementation is very efficient, i.e. fast in execution with low memory requirements
   - Closed mathematical form aids visualisation, modification and analysis
2. Analysis of the obtained models
   - Sensitivity analysis (Sobol, mutual information)
   - Frequency response
   - Graphical methods
   - Other?

Robot Modelling, Identification and Characterisation: Summary

- Objectives

  1. Theory
  2. Modelling of robot-environment interaction
  3. Identification and characterisation of robot-environment interaction
  4. Tools

Robot Modelling, Identification and Characterisation: Summary

- Objectives

  1. Theory
  2. Modelling of robot-environment interaction
  3. Identification and characterisation of robot-environment interaction
  4. Tools

- Approach

  – Model input-output relationships, using linear or non-linear polynomial models

Robot Modelling, Identification and Characterisation: Summary

- Objectives

  1. Theory
  2. Modelling of robot-environment interaction
  3. Identification and characterisation of robot-environment interaction
  4. Tools

- Approach

  – Model input-output relationships, using linear or non-linear polynomial models

- Advantages

  – Transparent
  – Compact
  – Analysable
  – Transferable

Robot Navigation: Conclusions

- Task-achieving navigation (e.g. self-localisation or route learning) is possible without symbolic representation of the environment

Robot Navigation: Conclusions

- Task-achieving navigation (e.g. self-localisation or route learning) is possible without symbolic representation of the environment
- It is possible to express these navigational competencies in closed mathematical form, through system identification

Robot Navigation: Conclusions

- Task-achieving navigation (e.g. self-localisation or route learning) is possible without symbolic representation of the environment
- It is possible to express these navigational competencies in closed mathematical form, through system identification
- The resulting models reveal
  - The relevance or irrelevance of individual sensors
  - The behaviour of the robot under specific operating conditions
  - The stability of behaviour
  - The interdependency between richness of sensory perception and integration of perception over time

Further Information

- Ulrich Nehmzow, *Scientific Methods in Mobile Robotics* Springer Verlag 2006

- http://cswww.essex.ac.uk/staff/udfn