

Core Knowledge of Number and Geometry

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Understanding Cognitive Systems

Attempts to build artificial cognitive systems of use to humans will benefit from understanding human cognition. This understanding, in turn, can be informed by studies of cognition in:



infants



children



animals



adults

- Studies of infants reveal the capacities at the foundations of all human knowledge.
- Probing children's cognitive development can reveal how human knowledge grows and changes with maturation and experience.
- Most of our cognitive capacities evolved before we did; studies of other animals often reveal them most clearly.
- Comparisons across cultures serve as a converging way of discovering universal, foundational human capacities and for exploring human flexibility and diversity.

A Substantive Proposal: Core Knowledge

At the foundations of human knowledge is a set of core systems for representing significant aspects of the environment:

- objects and their motions
- agents and their goal-directed actions
- places and their geometric relations
- sets and their approximate numerical relation

Core systems have signature limits: domain-specific, task-specific, encapsulated.

Core systems are shared by other animals, persist in adults, and show little variation by culture, language, or sex.



Three core systems: small numbers of objects, large numbers, geometry

Evidence: From studies of human infants, in relation to non-human animals and adults in far-flung cultures.

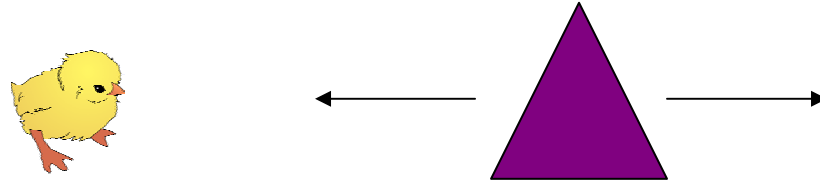


A question: How do humans go beyond core knowledge and construct uniquely human (and culturally variable) concepts and cognitive capacities?

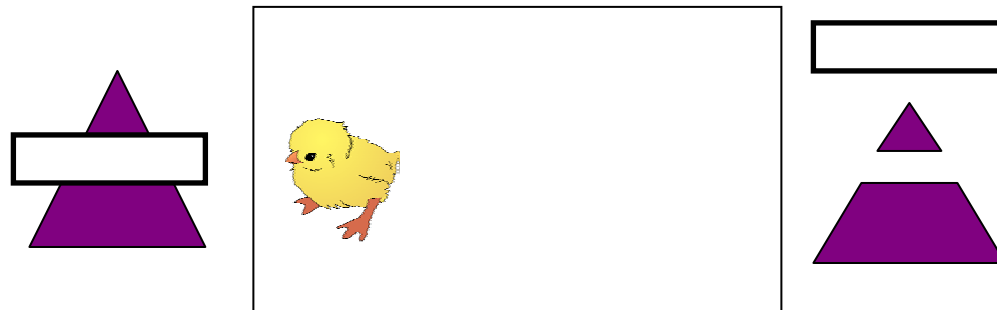


Studies of the origins of object perception: chicks!

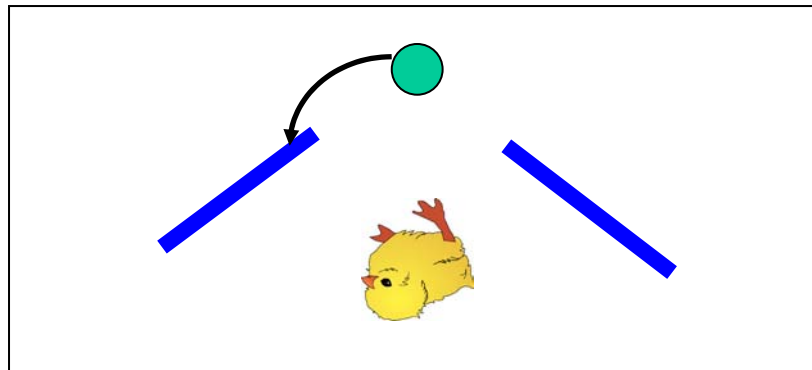
imprinting phase (day 1, home cage)



test phase (day 2, new cage)



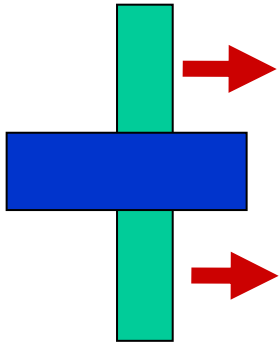
Or.....



Perception of
occluded objects
without visual
experience of
occlusion

(Regolin, Vallortigara and colleagues, 1995-present)

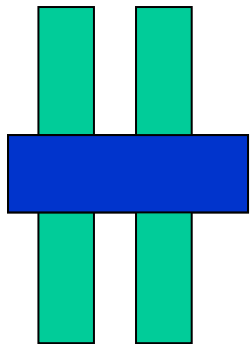
Object perception in human newborns: A brief history



Newborns: object failure
2 months: partial success
4 months: success

General conclusion: Humans learn to represent objects, unlike chicks (Slater et al, 1990; Mareschal & Johnson, 2002).

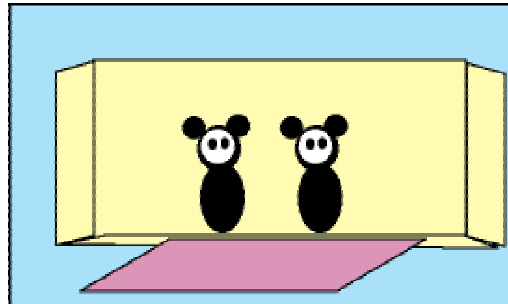
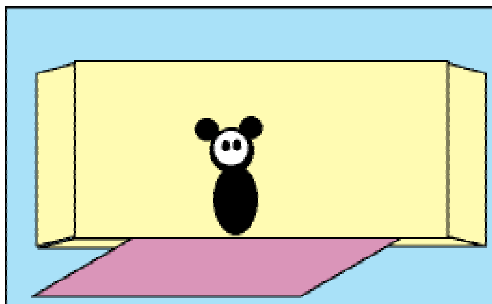
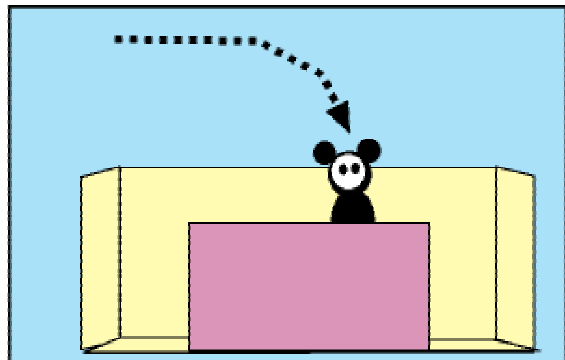
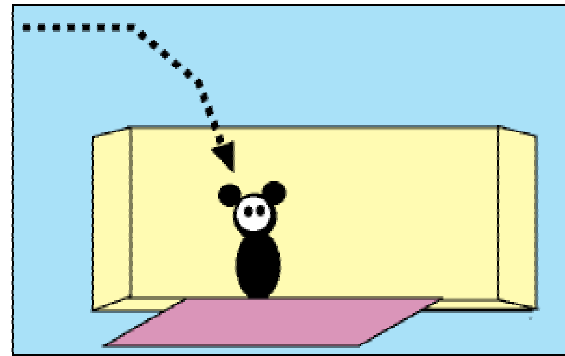
Alternatively: Human newborns may represent objects but have low sensitivity to motion (e.g., Wattam-Bell, Atkinson & Braddick).



Valenza, Leo, Gava & Simion (2005): High sensitivity to motion in the superior colliculus, which develops early. Works best for rapid or discontinuous motion.

Conclusion: perceptual completion in human newborns.

Addition and Subtraction of Objects



$1 + 1 = 2$ not 1 or 3

$1 + 2 = 3$ not 2

* $5 + 5 = 10$ or 5??

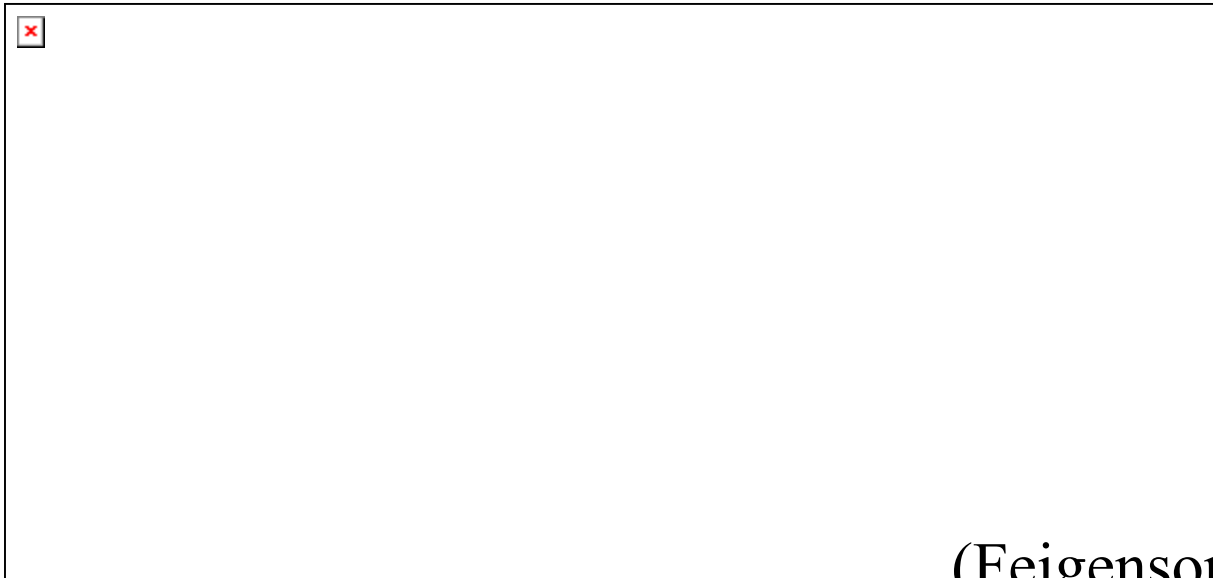
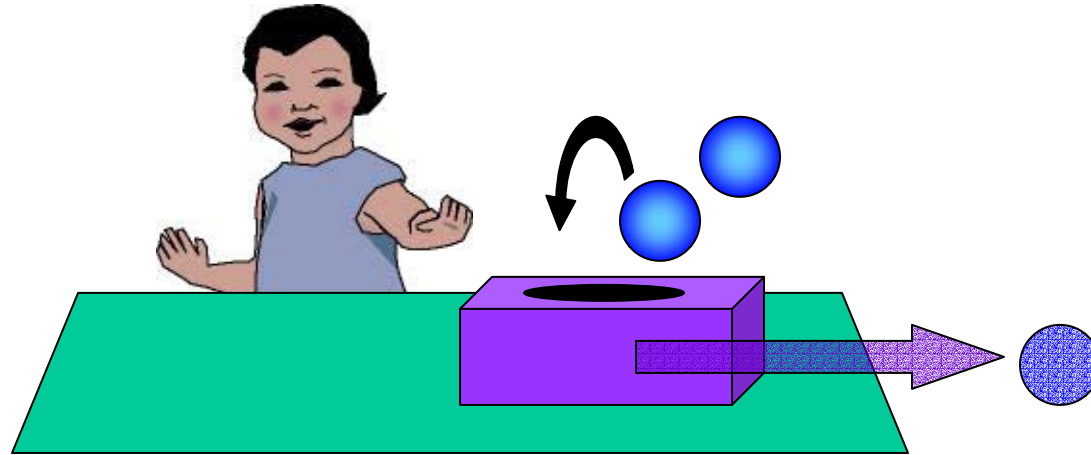
Set size limit: 3-4

Domain limit: entities that obey cohesion, contact, and continuity constraints

(Wynn, 1992)

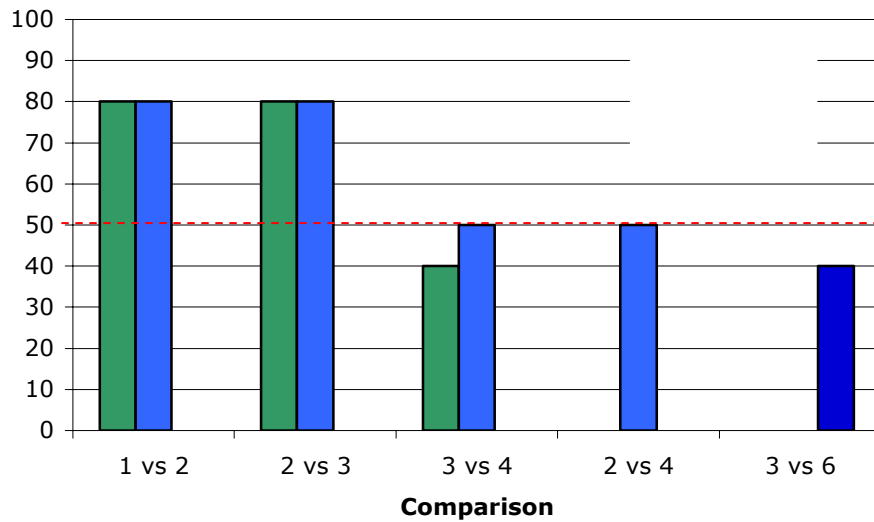
*

Converging evidence: The box search task (12 & 14 months)



(Feigenson & Carey, 2003)

More converging evidence: the locomotor search task (10 & 12 months)



(Feigenson, Carey & Hauser, 2002)

Core Knowledge of Objects: Signatures

1. Cohesion
2. Contact
3. Continuity
4. Set size limit (3)

Common signature limits across different behavioral tasks suggest a single representational capacity.

These signatures can be used to test for the capacity in other animals and at other ages.

Object representations in monkeys



Like infants, monkeys represent small numbers of objects (many Hauser papers, 1996-present).



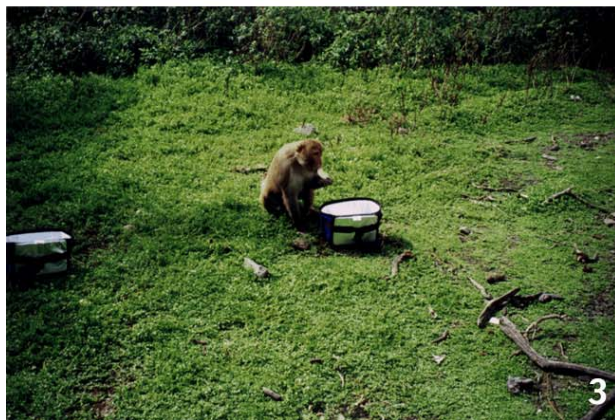
Monkeys' representations show the same signature limits as infants:

Cohesion

Contact

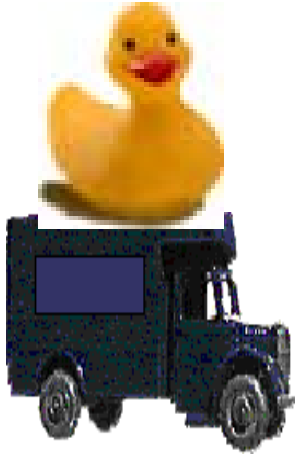
Continuity

set size limit (4).

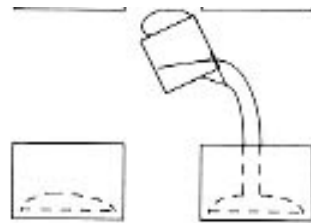


A common system of representation over primate evolution.

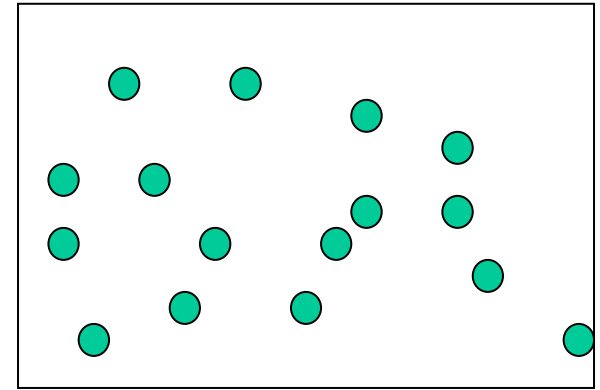
Object perception in adults: Same signatures?



“duck,” “truck”

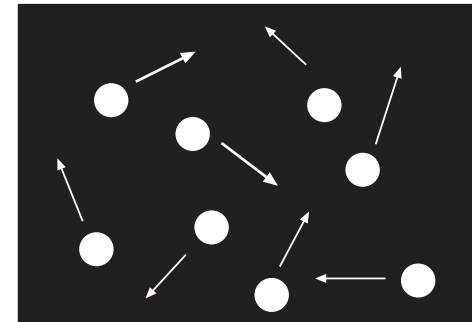
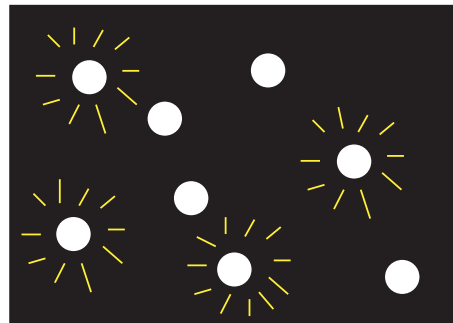


“two sand piles”



“exactly sixteen”

multiple object tracking:



Object representation in adults: The multiple object tracking task

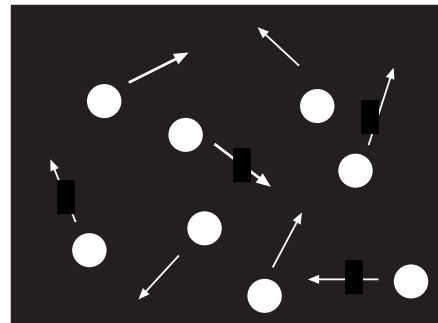
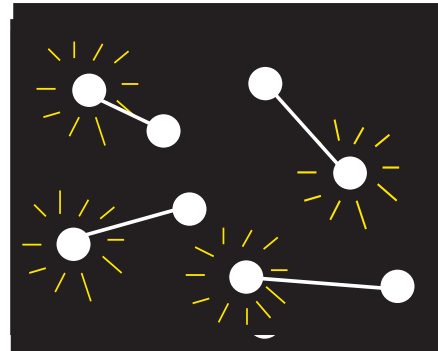
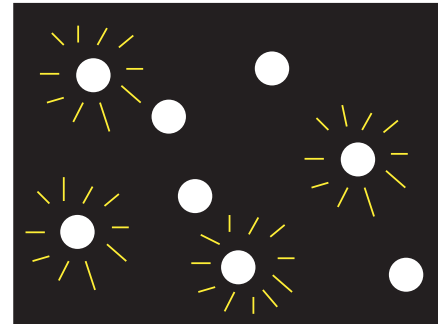
MOT in adults shows all the signatures of infants' object perception:

Set size limit (3-4)

Cohesion

Contact

Continuity



Scholl, Wynn,
Mitroff,
VanMarle and
others (2000s)

A common system of representation over human ontogeny.

Object representation across cultures: The Piraha

The Piraha are cognitively and linguistically very different from all other known peoples:

- no number words, counting, or grammatical number.
- no color terms.
- no distinction of past vs. present.
- no embedding (e.g., relative clauses).

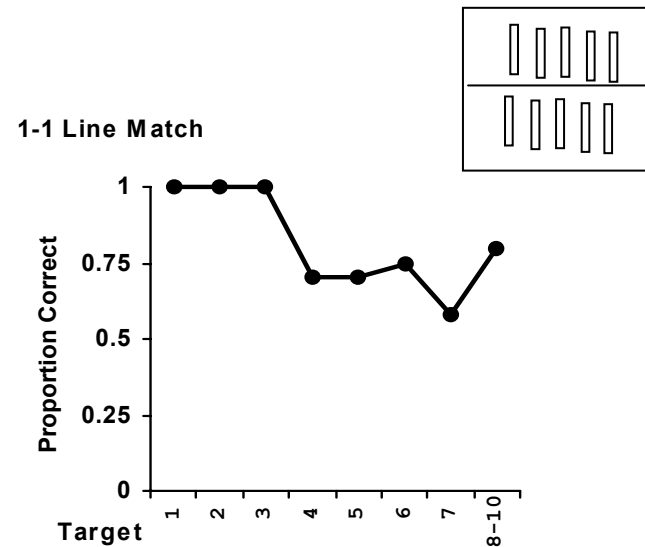
But: they distinguish cohesive objects from non-cohesive substances (example: “many foreigners” vs. “much manioc meal”)



(Everett, 2005)

More on the Piraha

And, they show the set size signature (3)



(Gordon, 2004)

A common system of representation over human cultures.

Core knowledge of small numbers: Summary



Human infants (including newborns), non-human animals, and human adults in diverse cultures represent small numbers.



Their representations of objects show four signature limits:

cohesion

contact

continuity

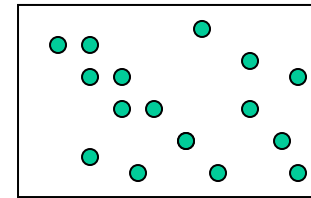
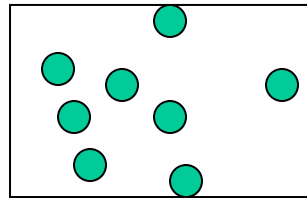
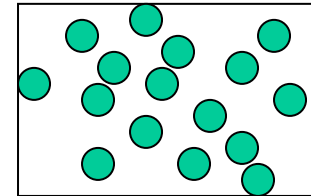
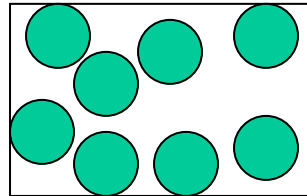
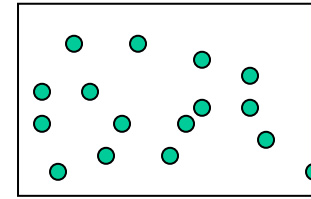
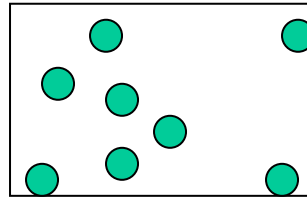
set size limit of 3-4.



Evidence for core knowledge.

System 2: Core Knowledge of Large, Approximate Numbers

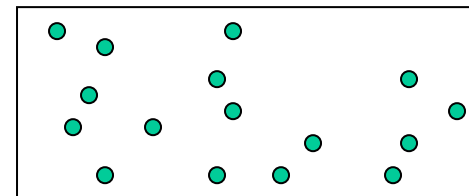
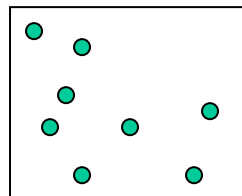
(6-month-old infants)



(...)

(...)

Findings: longer looking at new number.



(Xu & Spelke, 2000)

Signatures of infants' performance

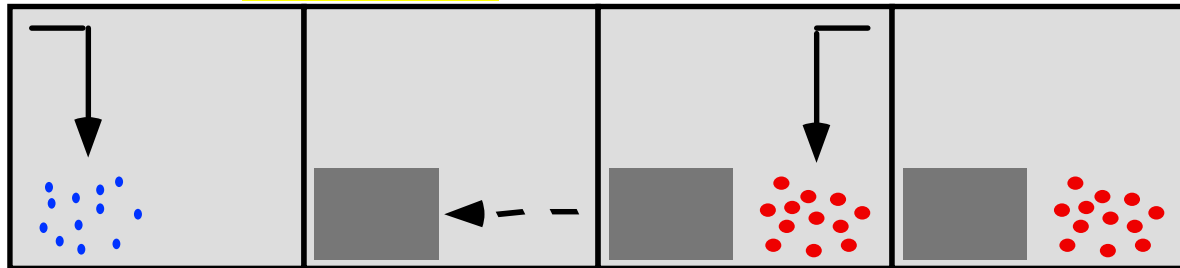
1. Performance depends on the ratio of the set sizes:

6 months:	8 vs. 16	*8 vs. 12
	4 vs. 8	*4 vs. 6
	16 vs. 32	*16 vs. 24
9 months:	8 vs. 12	*8 vs. 10

2. Modality- and format invariance: Same limits with visual-spatial arrays (dots), visual sequences (jumps), and auditory sequences (beeps, syllables); cross-modal matching
3. Addition & subtraction (with simultaneously available sets: McCrink & Wynn, 2005)

Visual comparison and addition in preschool children

Preschool **comparison** task: more blue dots or more red dots?



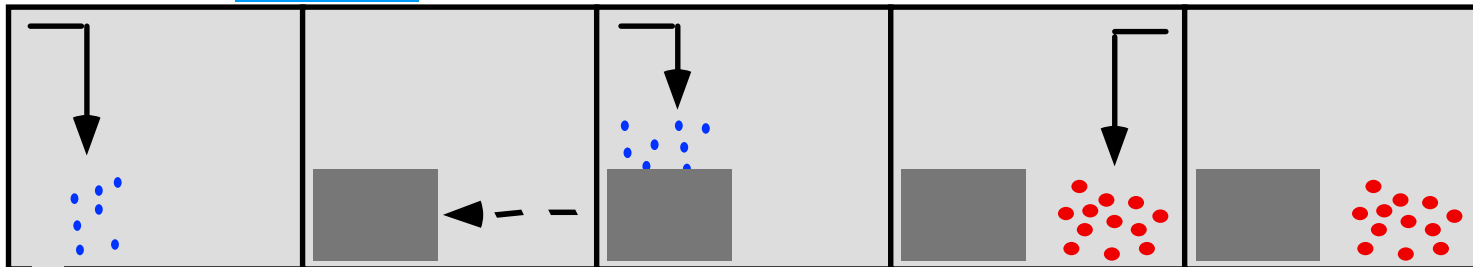
"Look, here come some blue dots!"

"Now they're being covered up!"

"And here come some red dots!"

"Are there more blue dots, or more red dots?"

Preschool **addition** task: more blue dots or more red dots?



"Look, here come some blue dots!"

"Now they're being covered up!"

"Here come some more blue dots. Now they're ALL back there!"

"And here come some red dots!"

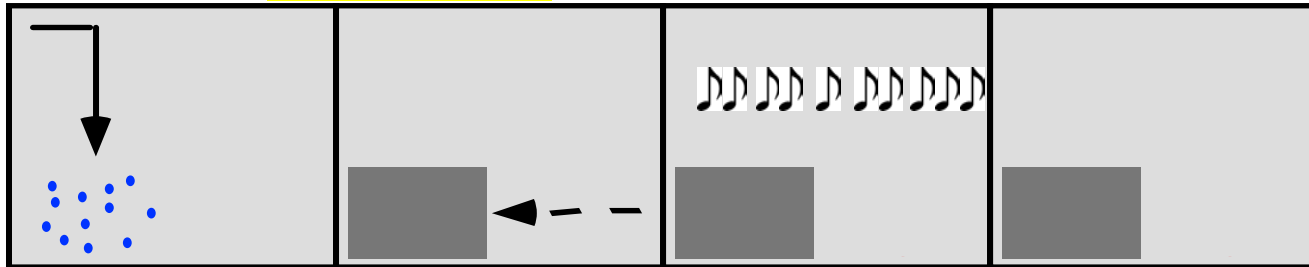
"Are there more blue dots, or more red dots?"

Ratios: high (4:7)
medium (4:6)
low (4:5)

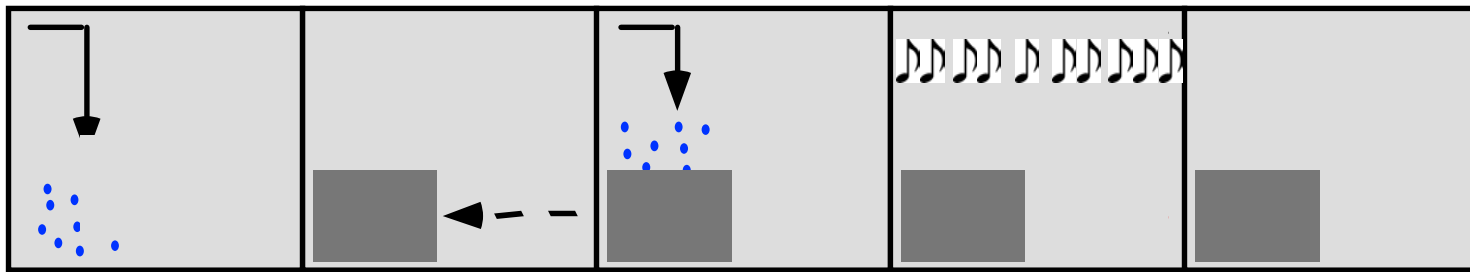
Barth, LaMont, Lipton & Spelke (2005)

Cross-modal comparison and addition in preschool children

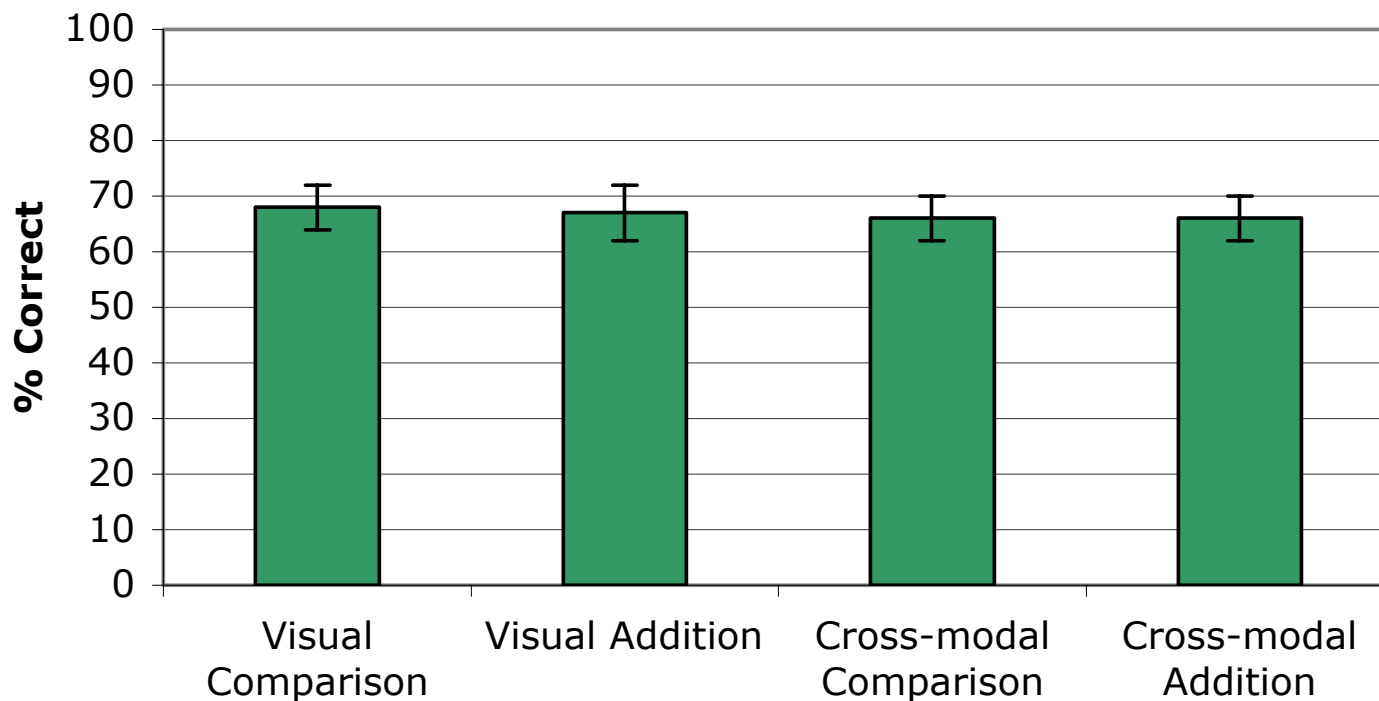
Comparison



Addition



Visual vs. cross-modal comparison & addition (5-year-old children)



Children show equal accuracy on visual and cross-modal tasks. These abilities emerge before children learn arithmetic: not dependent on symbolic number knowledge.

Spontaneous large number discrimination in tamarin monkeys



Method: similar to studies with infants.

Signatures:

1. ratio limit

success

4 vs. 6

8 vs. 12

failure

*4 vs. 5

*8 vs. 10

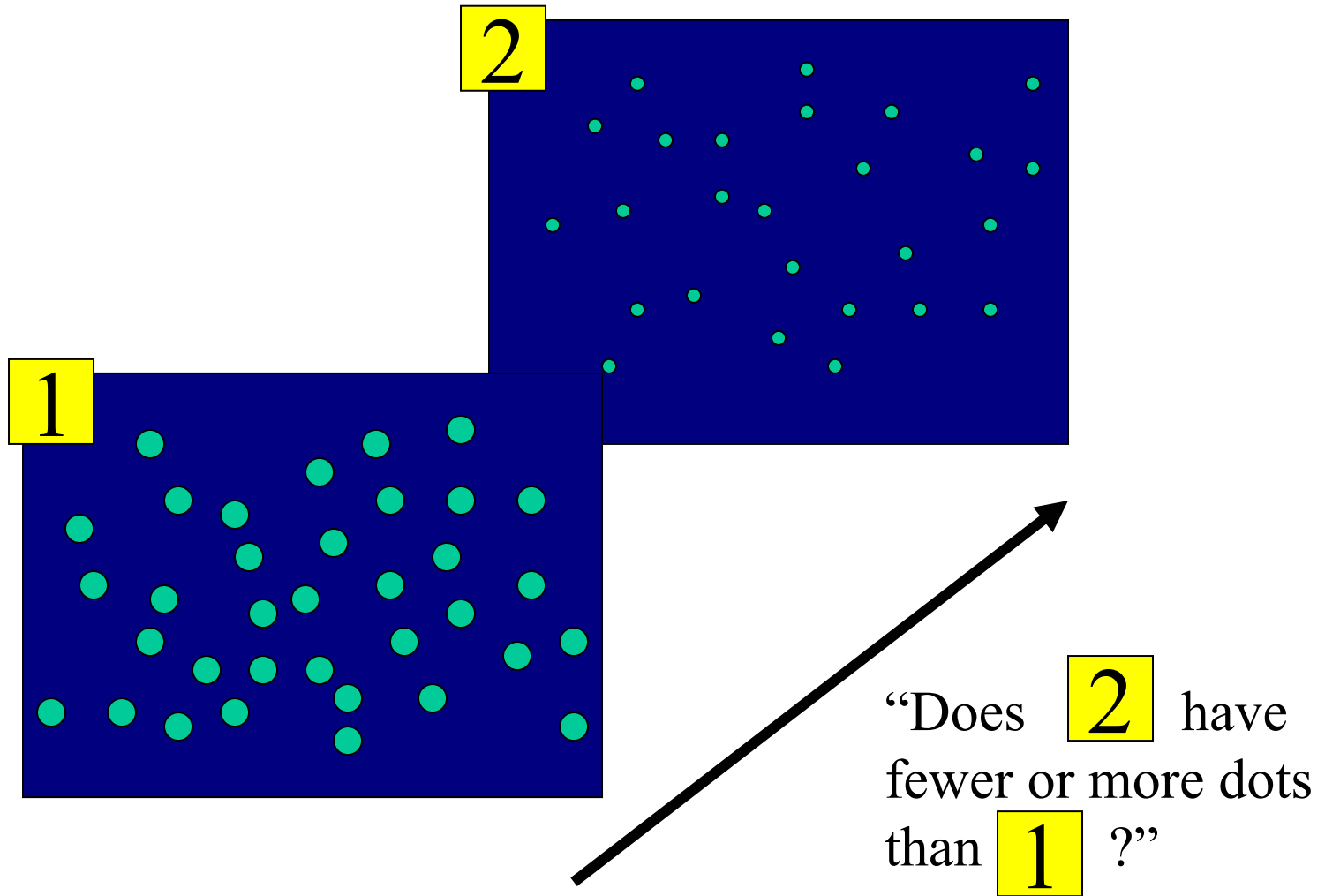
2. format invariance

3. addition & subtraction with simultaneous sets

A common system of representation over primate evolution.

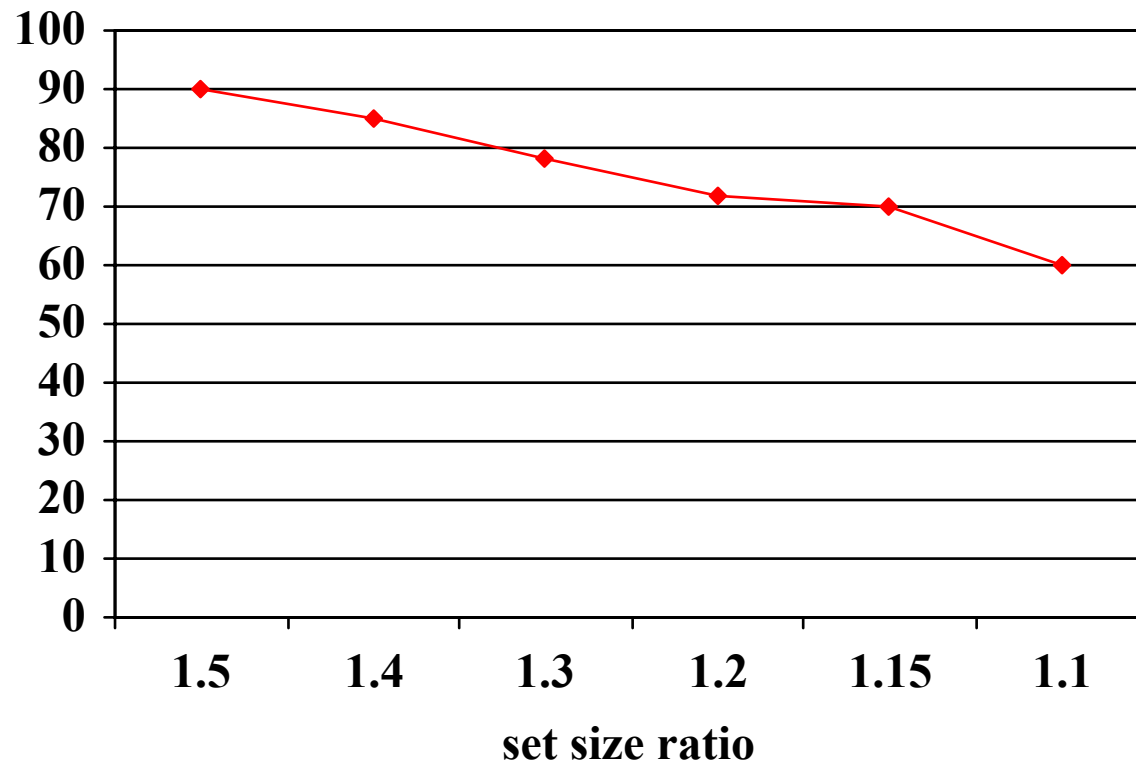
(Hauser, Tsao, Garcia & Spelke, 2003; Flombaum & Hauser, 2005)

Numerosity discrimination in human adults



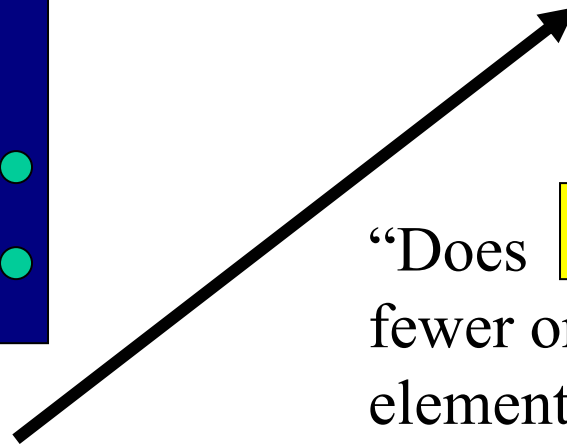
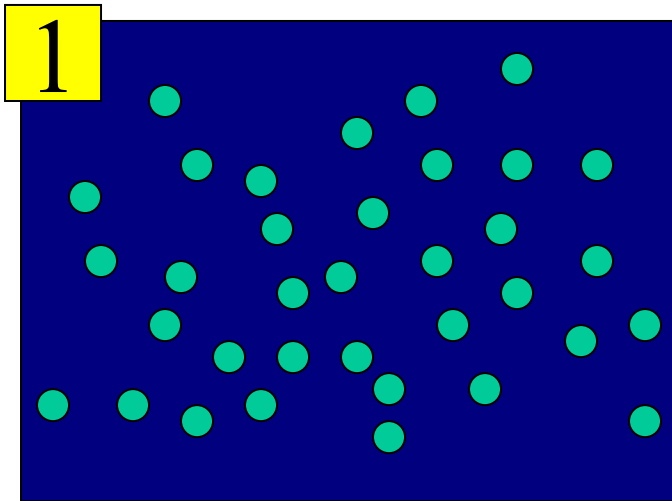
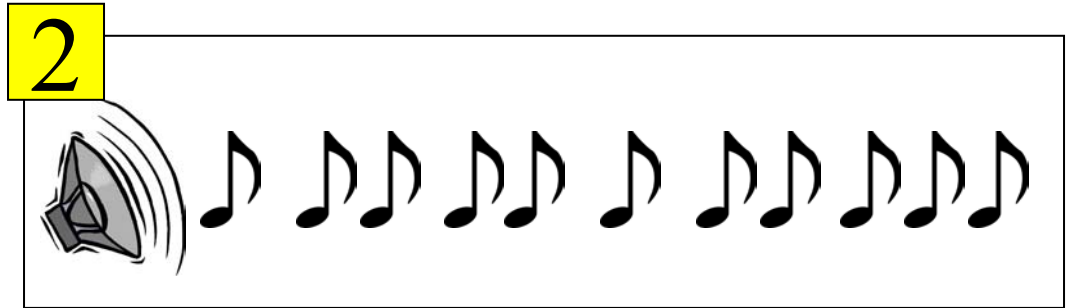
(Barth, Kanwisher & Spelke, 2001)

Adults' numerical comparisons with visual arrays

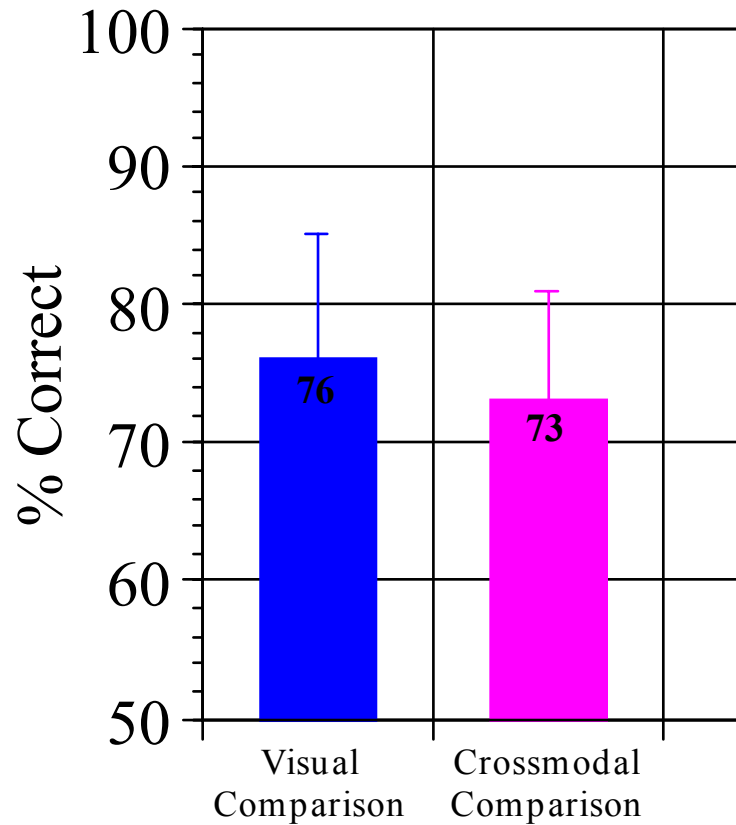


Accuracy varies with set size ratio, independent of absolute set size.

Format invariance in adults' large number representations?



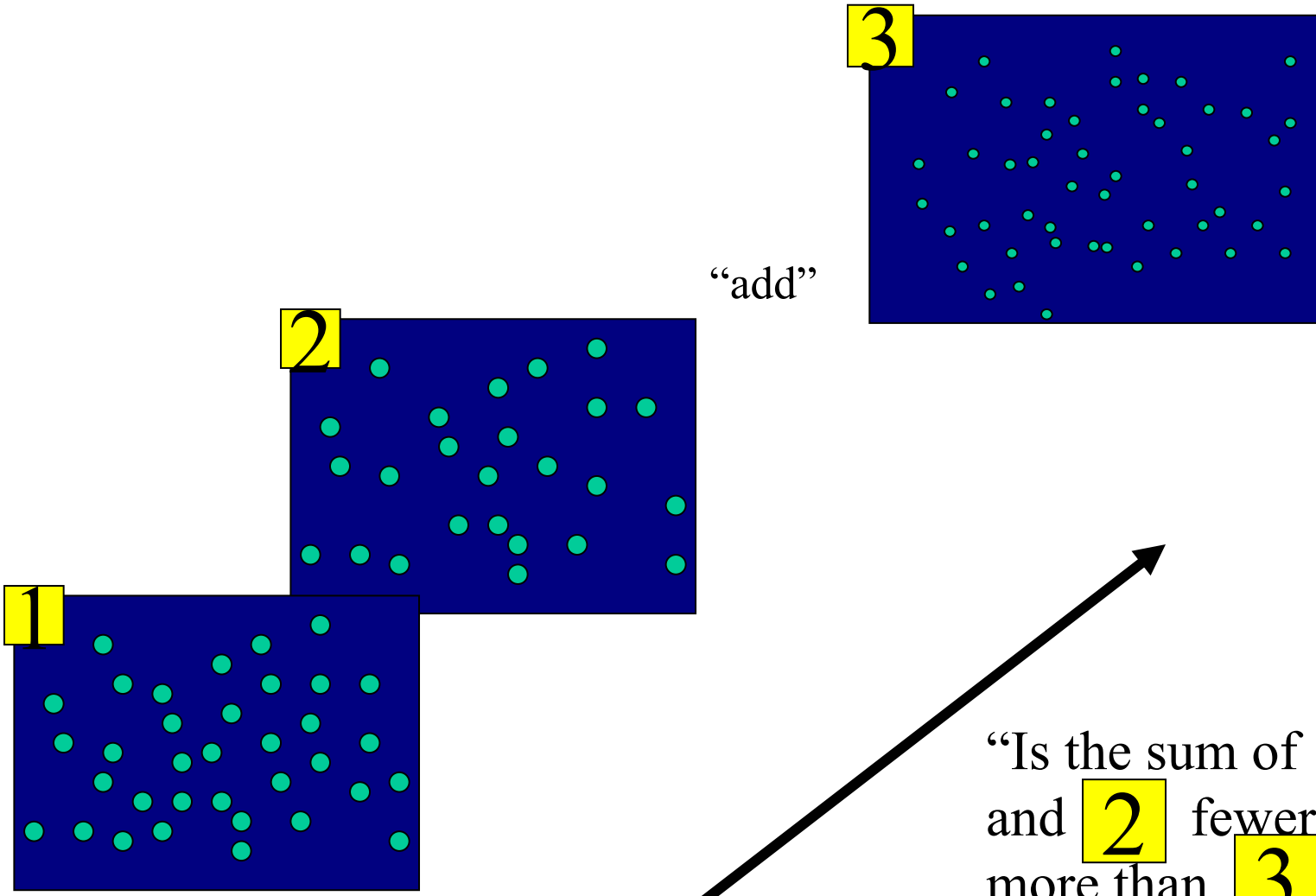
“Does 2 have fewer or more elements than 1?”



Cross-modal comparisons are almost as accurate as comparisons within the visual modality alone.

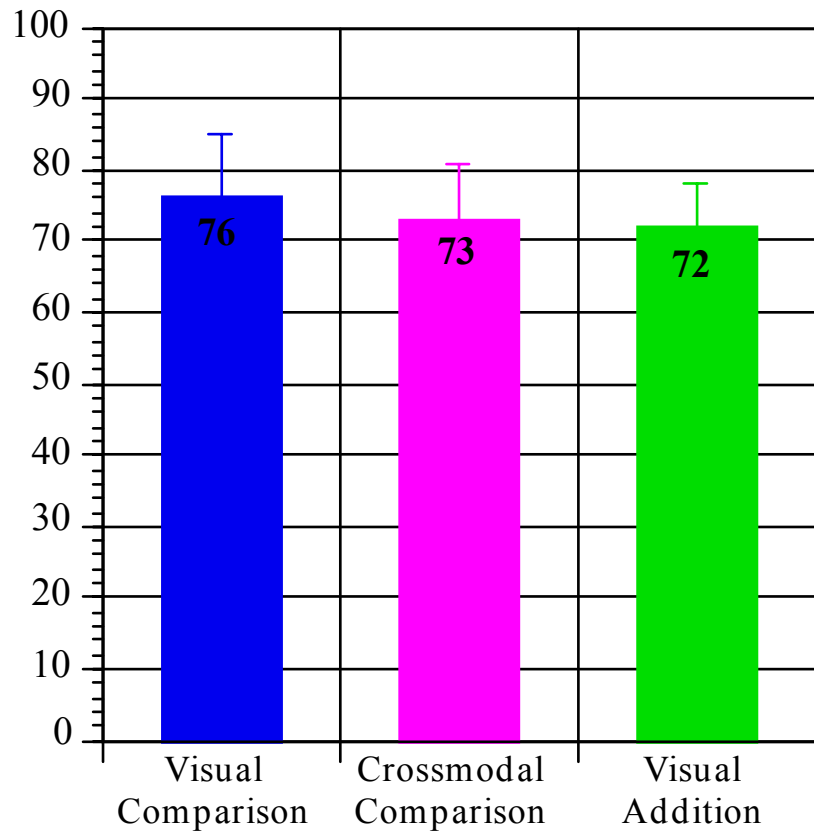
(Barth, Kanwisher & Spelke, 2001)

Addition of visual arrays



“add”

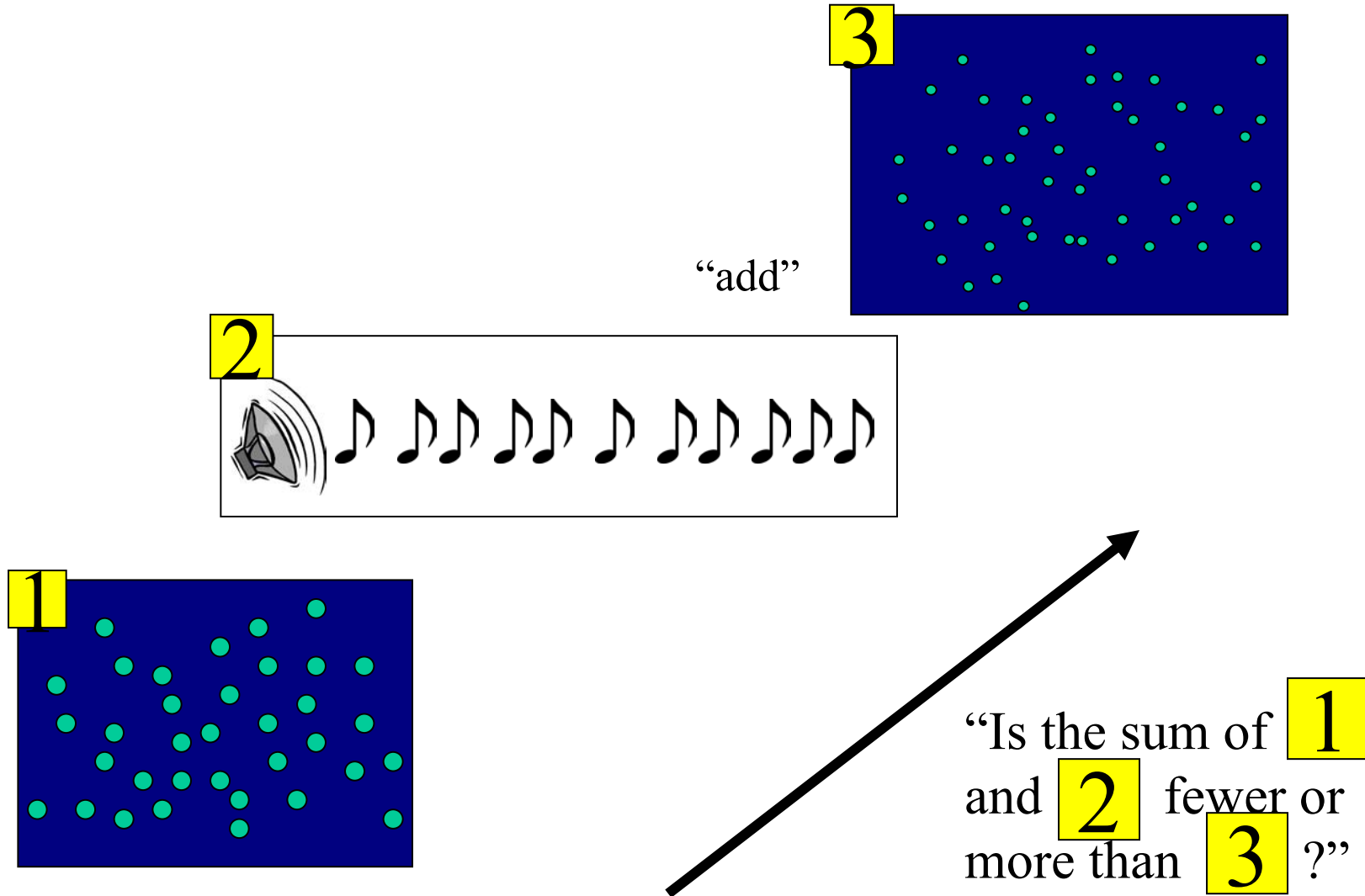
“Is the sum of **1** and **2** fewer or more than **3**?”

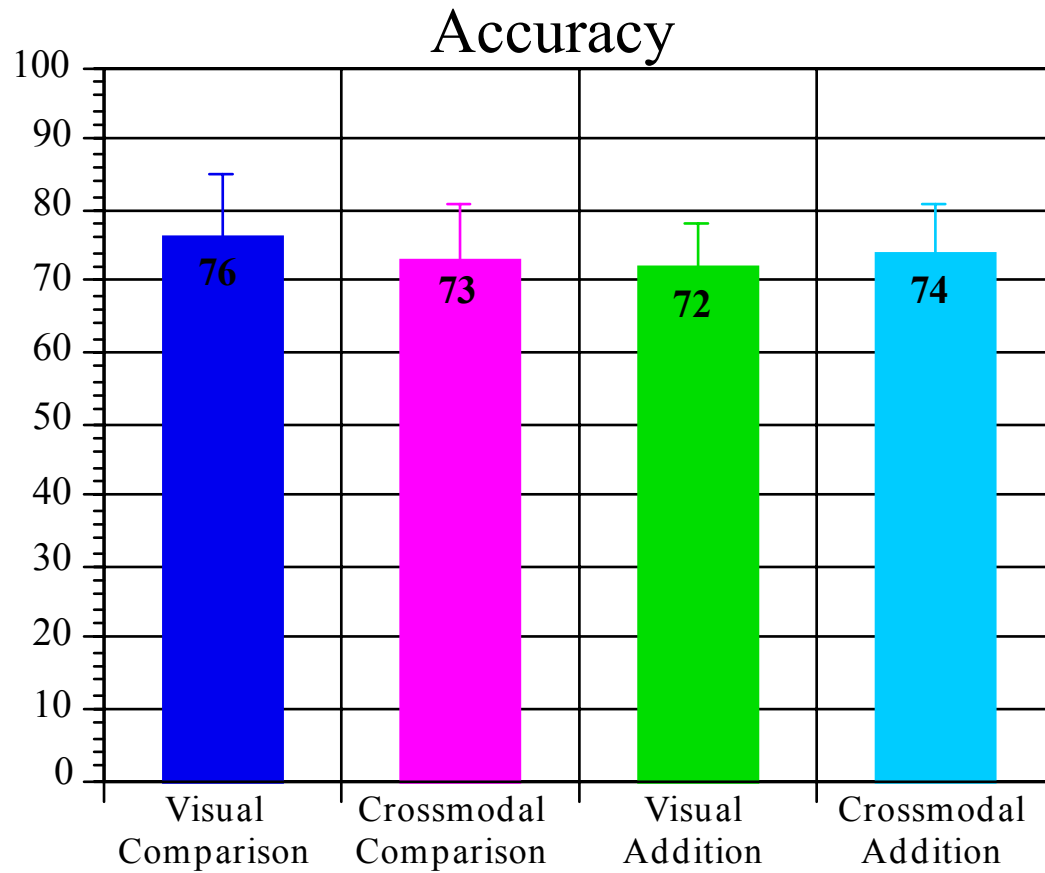


Addition is almost as accurate as comparison.

(Barth, Kanwisher & Spelke, 2001)

Cross-modal addition



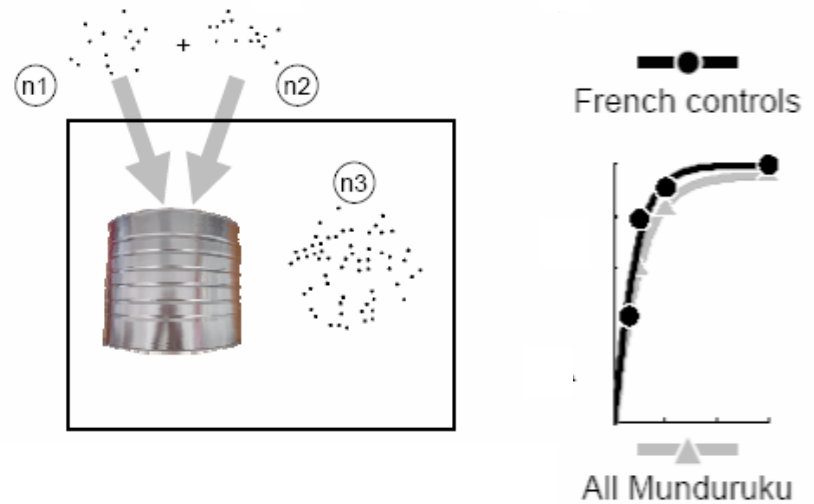


(Barth, 2001)

Adults show all the signatures of infants' and monkeys' large number representations.

Are these representations universal?

The Mundurucu

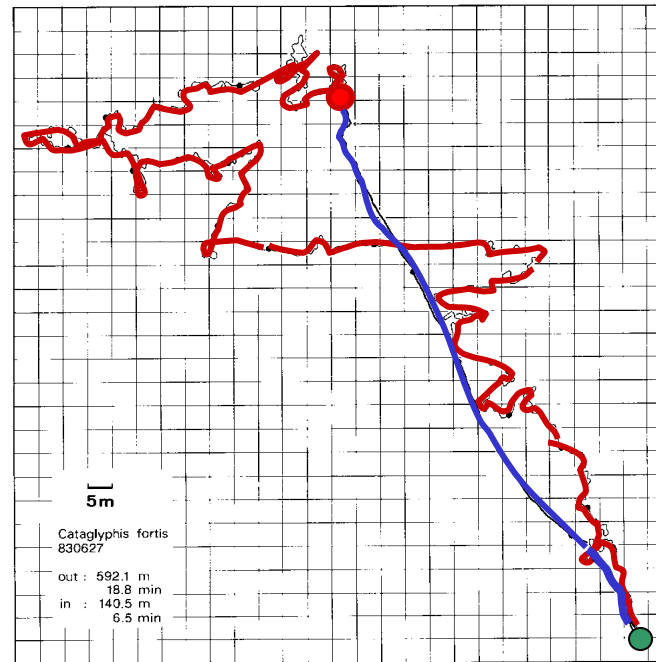


(Pica, Izard, Lemer & Dehaene, 2004)

System 3: Core Knowledge of Geometry



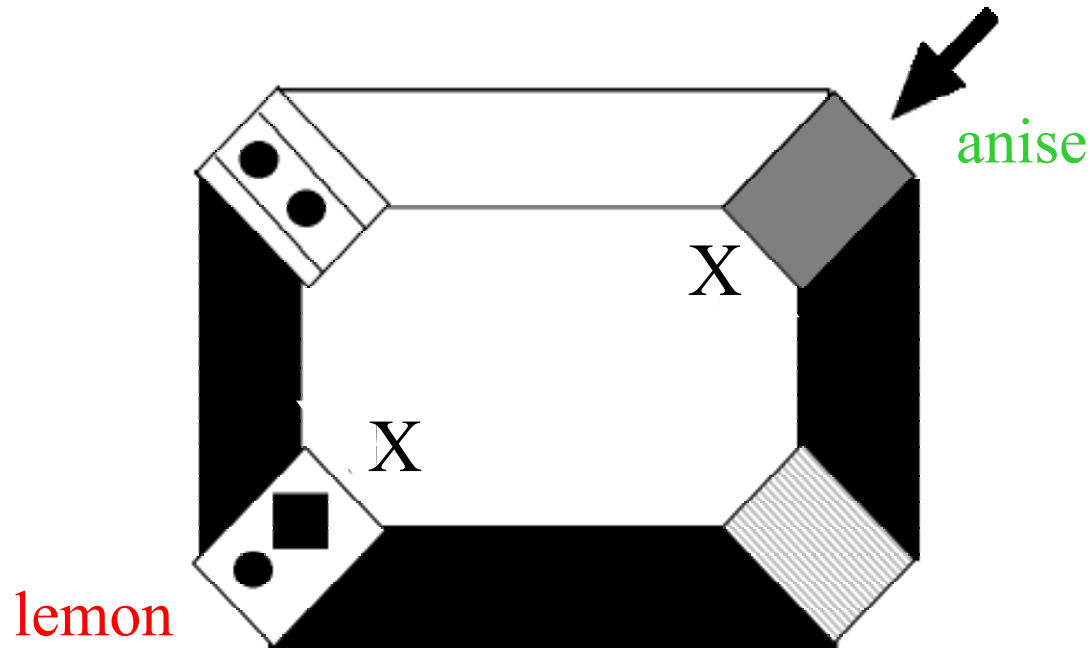
Rat at the end of a T-maze.
After Tolman, 1961.



Outward and **homeward** paths of an individually foraging desert ant, southern Tunisia.
After Wehner & Wehner, 1990

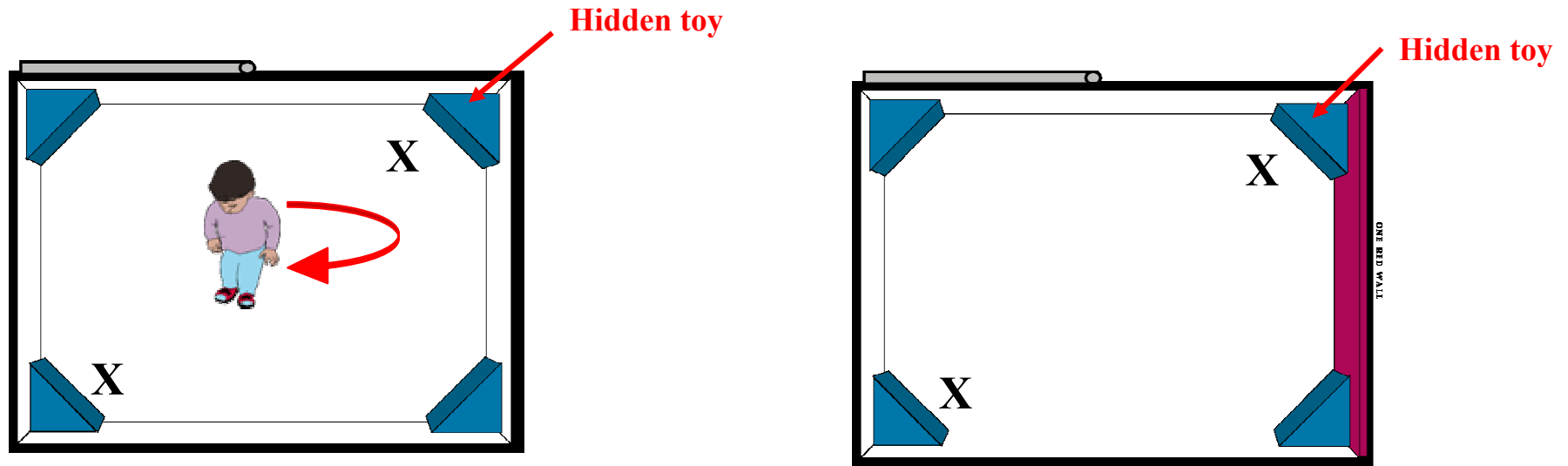
Reorientation in rats

(Cheng, 1986, Gallistel, 1990)



Rats reorient by the shape of the surrounding surface layout.
A signature limit: Rats fail to reorient by odors, patterns, & other features of the room, even though they perceive and remember these features navigate by them in other ways.

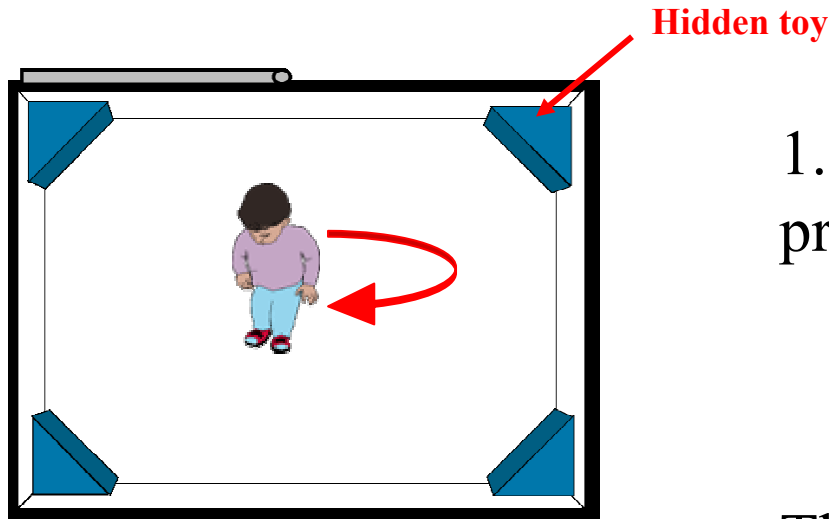
Reorientation in young children (18-24 months)



Children reorient by the shape of the room.

Children show the same performance limits as in rats, birds, fish, and other animals: A common system over phylogeny.

Key Properties of this System of Representation



1. Sensitivity to Euclidean geometric properties of the surface layout:

distance

sense

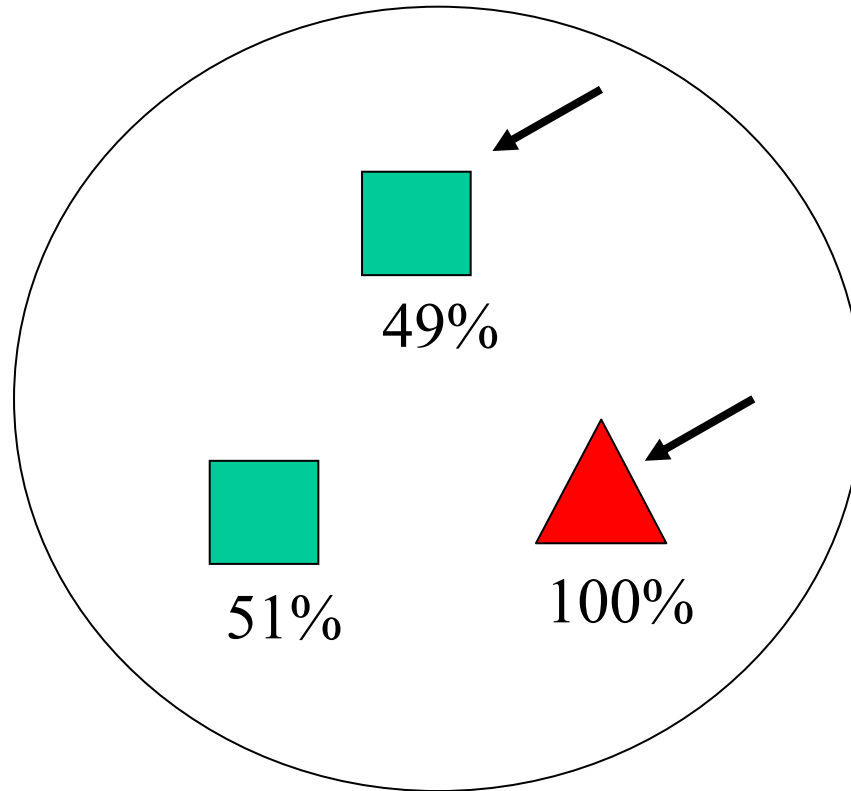
angle

These properties apply to surfaces of all sorts.

2. No reorientation by non-geometric properties of the layout (wall color, objects), though those properties are detected & used as landmarks.

(Hermer & Spelke, 1996; Gouteux & Spelke, 2001; Learmonth, Newcombe & Nadel, 2001)

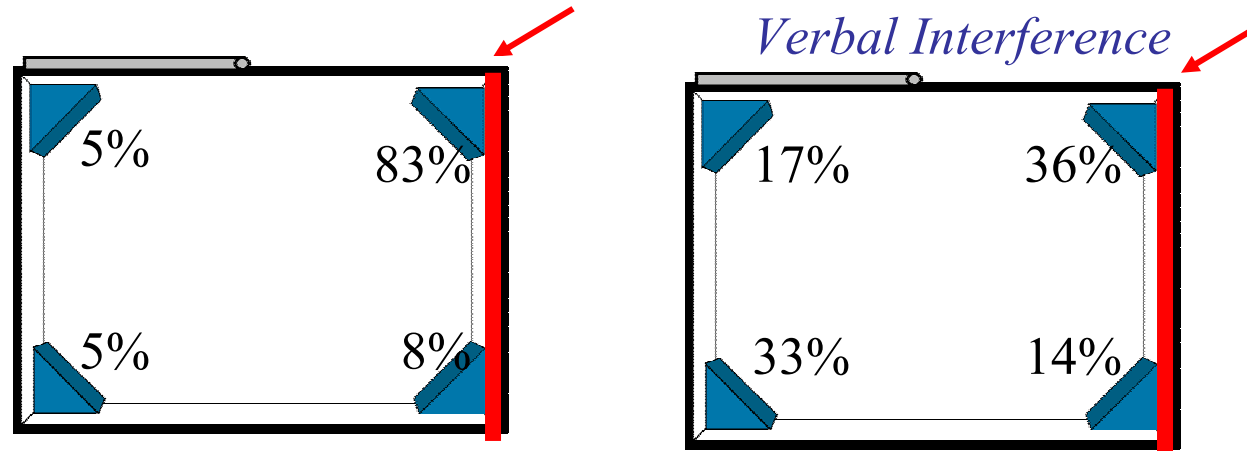
An example



Children remember the distinctive landmark but do not reorient by it.

Children do not combine geometric & landmark information flexibly:
*at the triangle, right of the long wall, *right of the triangle.*

Is this system present in human adults?



Adults, unlike children and rats, respond both to room shape and to the red wall. They also can describe the object's location in words. But what if they are given an interfering verbal task?

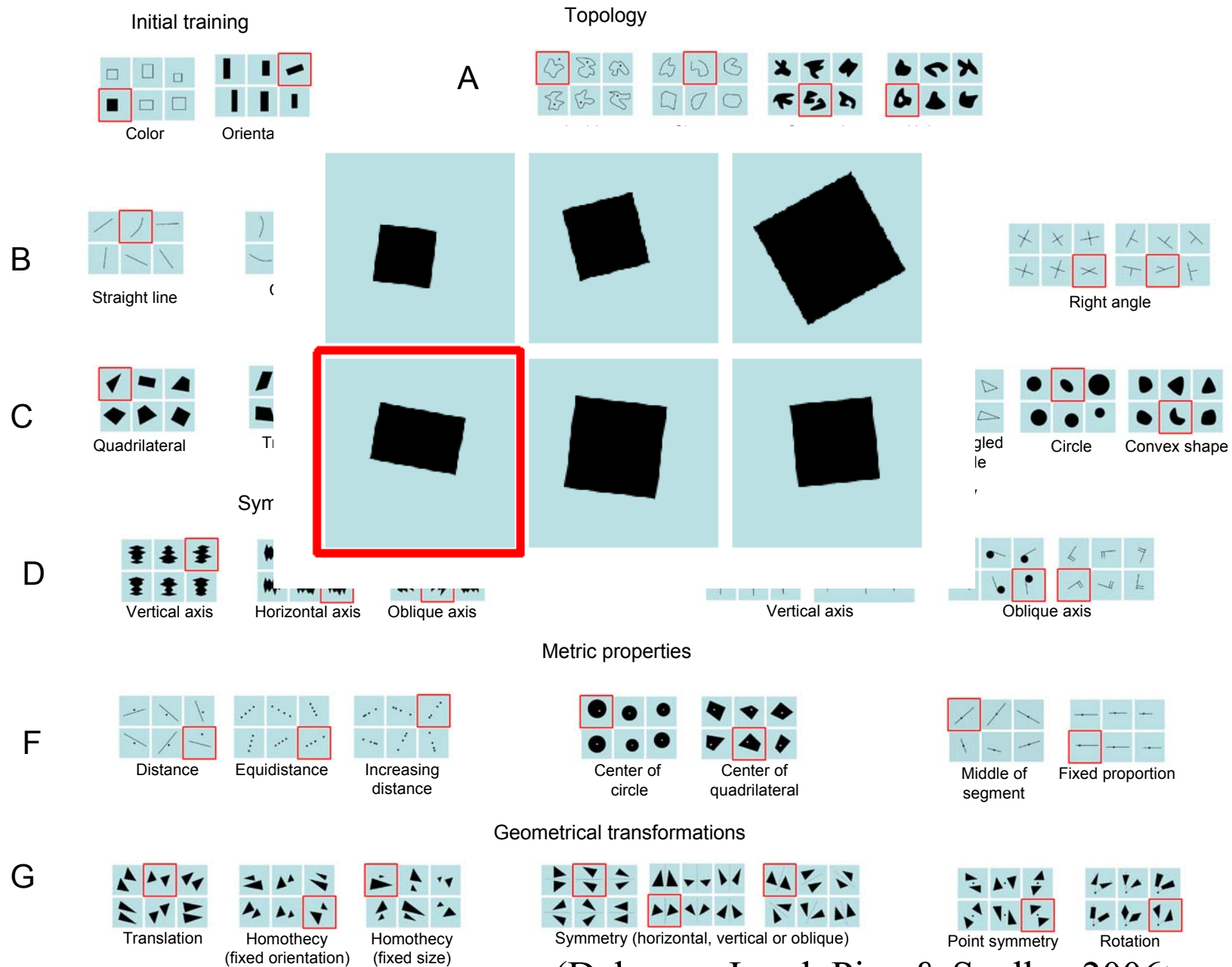
With interference, adults become like rats.

A common system over human ontogeny

(Hermer-Vazquez, et al., 1999; Ratliff & Newcombe, 2005)

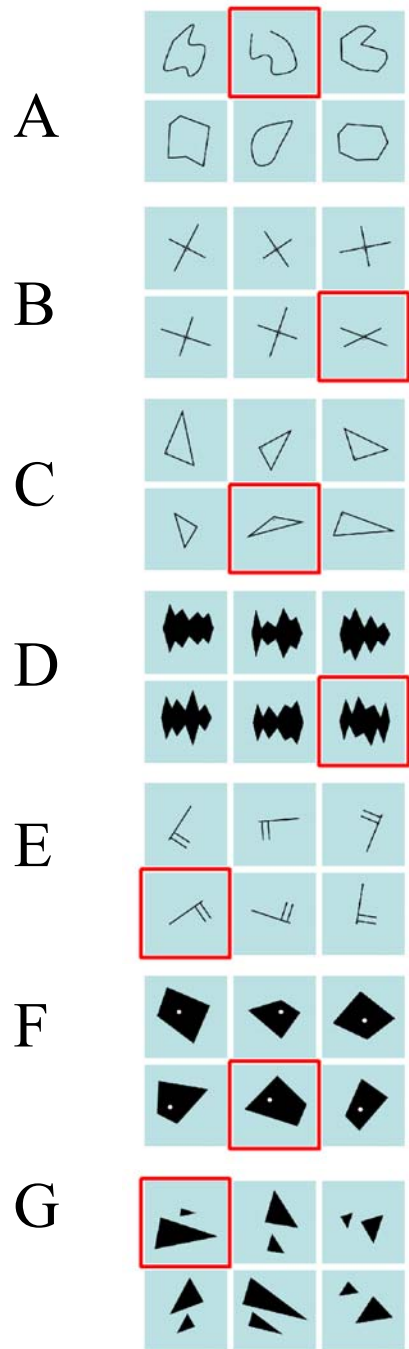
Testing the Breadth of Sensitivity to Geometry

Testing the Breadth of Sensitivity to Geometry

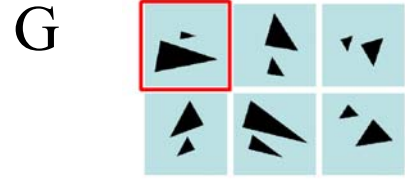
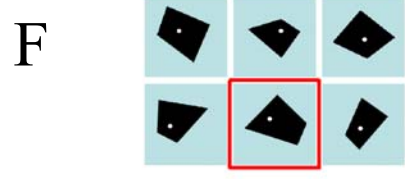
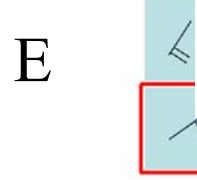
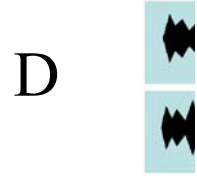
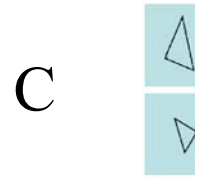
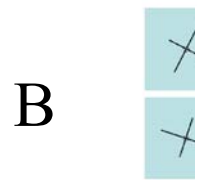


(Dehaene, Izard, Pica & Spelke, 2006)

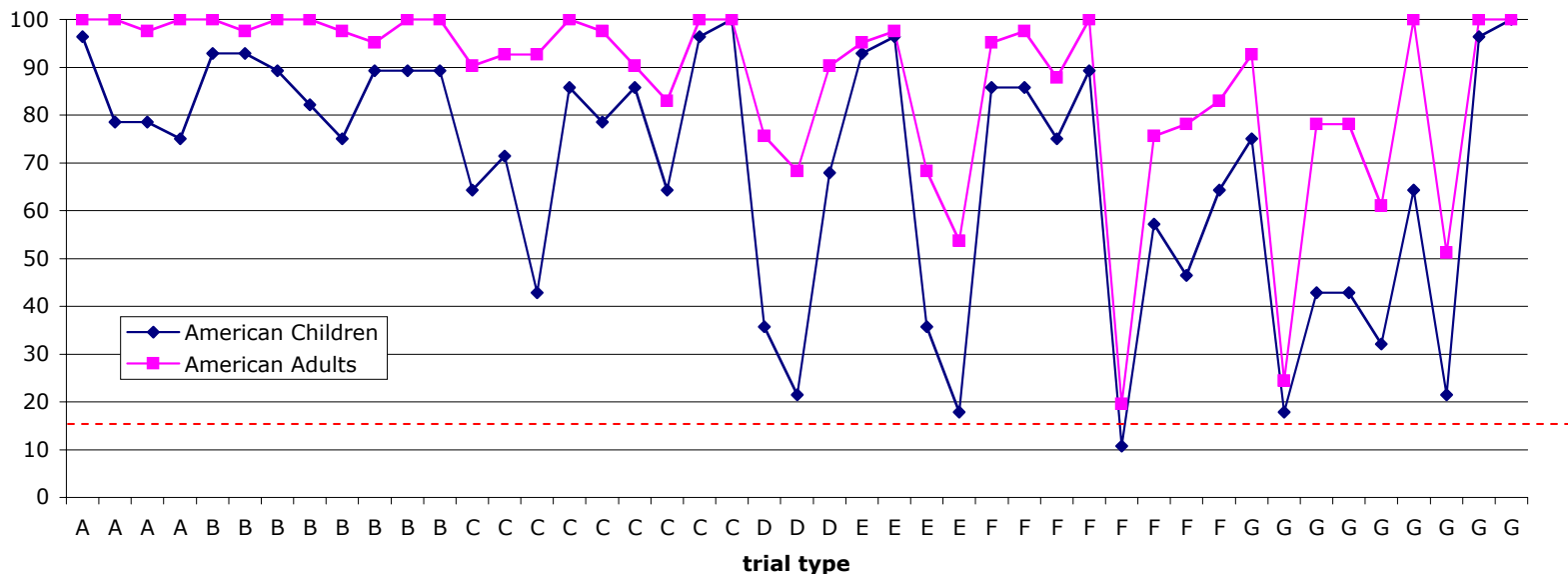
Sensitivity to Geometry



Sensitivity to Geometry



Performance by item: American Children & Adults



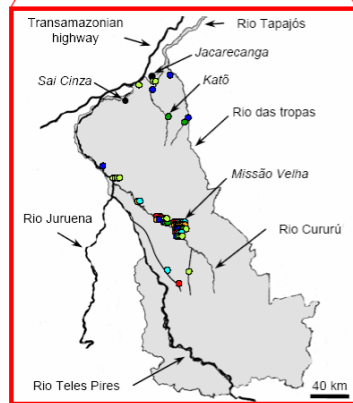
$r=.879, p \ll .001$

Improvement with age, but a strikingly similar pattern of performance.

Are geometric concepts universal across humans? studies of the Mundurukú



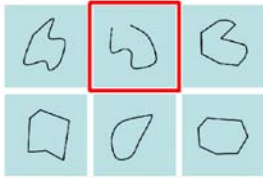
Mundurukú Territory



Adults	Children	
● n=9 (55.5 y)	● n=9 (4.7 y)	monolingual, no instruction
● n=10 (59.3 y)		bilingual, no instruction
	● n=7 (8.6 y)	monolingual, with instruction
● n=7 (38.7 y)	● n=13 (9.6 y)	bilingual, with instruction



(Dehaene, Izard, Pica & Spelke, 2006)



Sensitivity to geometry in the Mundurucu

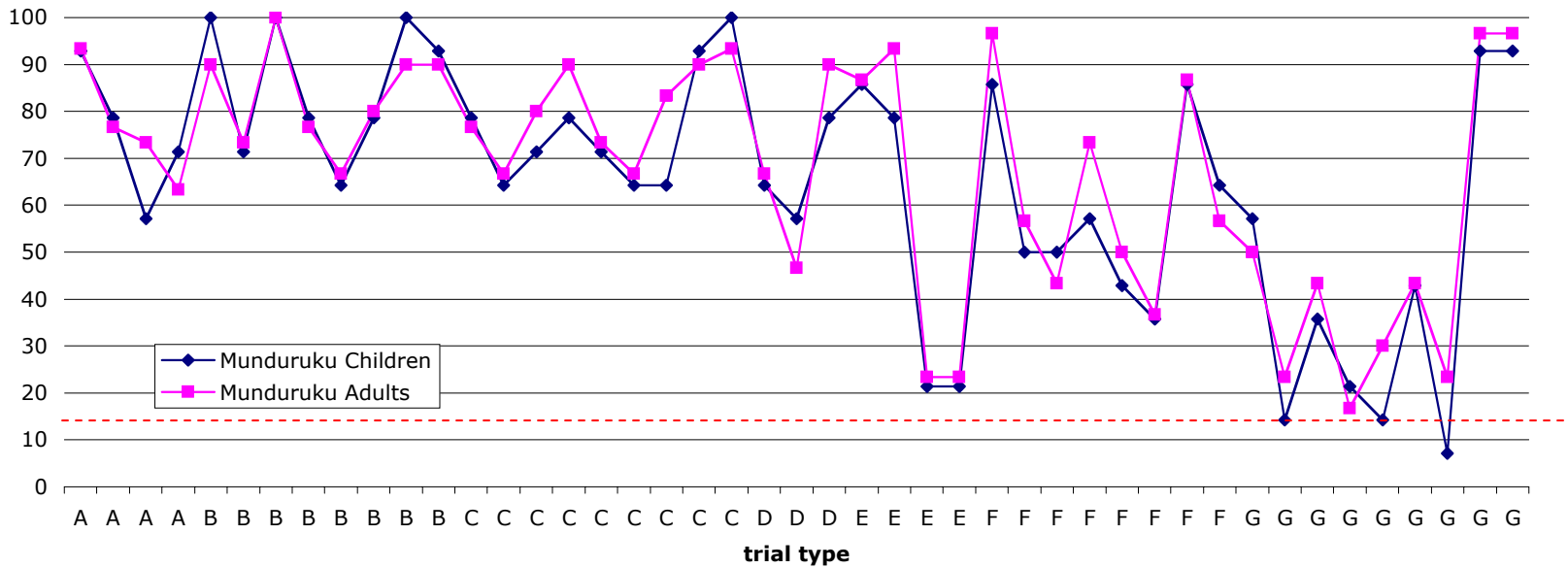
A



B

Performance by item: Mundurucu Children & Adults

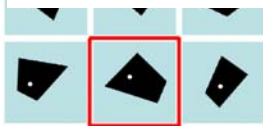
C



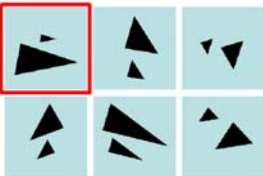
D

E

F



G

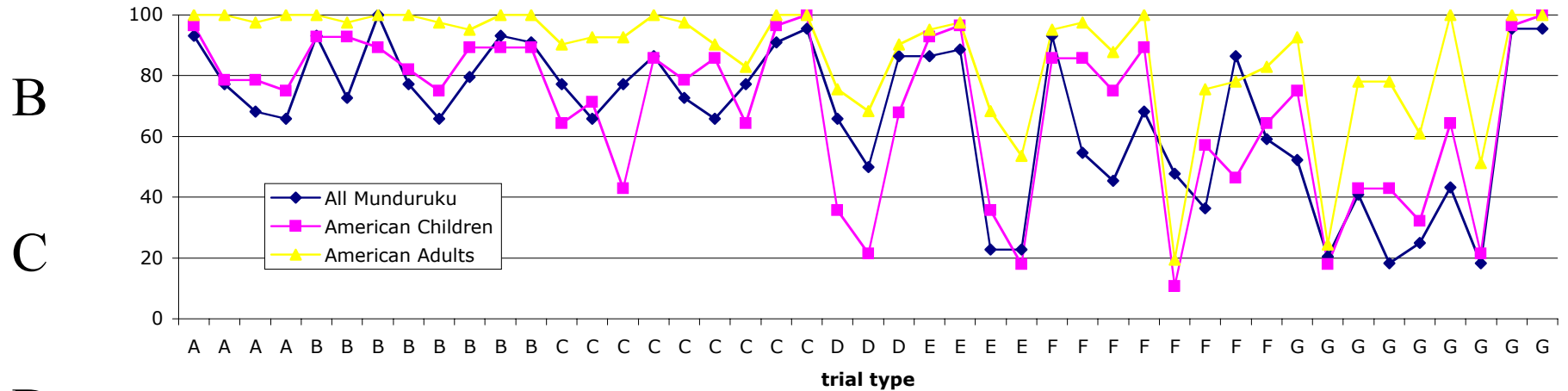




Sensitivity to geometry in two cultures

A

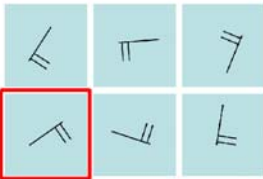
Performance by item: Mundurucu & American Children and Adults



D



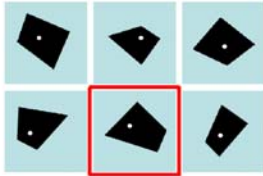
E



Mundurucu & American children: $r=.778, p<.001$

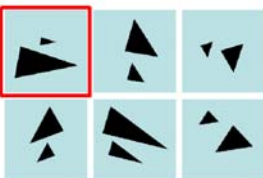
Mundurucu & American adults: $r=.705, p<.001$

F



Some effects of culture/education, but considerable invariance in geometrical representations across these different cultures.

G



Core knowledge of Geometry: Summary

Human infants and young children have a core system for representing the shape of the surrounding surface layout.

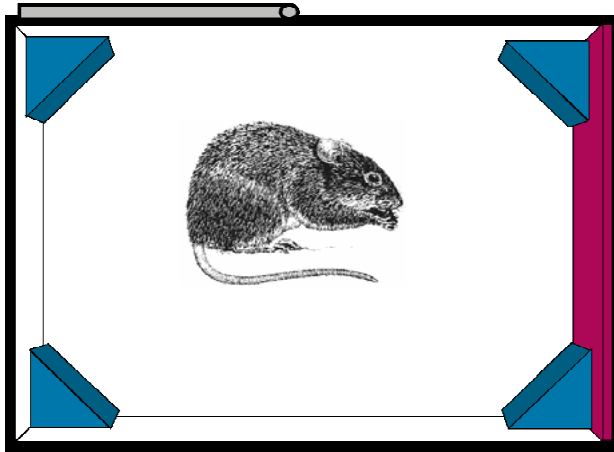
The system is shared by a variety of non-human animals including fish, birds, rats, and monkeys.

Its core constraints: Euclidean geometry, encapsulation against other non-geometric information.

Sensitivity to Euclidean relationships increases over development but shows a largely invariant pattern across age.

Sensitivity to Euclidean relationships varies somewhat across cultures, but again with a strongly invariant pattern.

Three core knowledge systems



1. Objects
2. Large numbers
3. Geometry



How do humans develop complex cognitive skills?

prolific tool use

cooking & food production

history & geography; politics &
economics

religion & taboos

music, dance, visual arts

baseball, theater, gossip...

mathematics & science



Limits of core knowledge

Infants and animals lack encyclopedic knowledge of object kinds.

Infants and animals can't represent "exactly seven"

Infants and animals can't combine landmarks and geometry productively.

How do humans go beyond core knowledge and construct concepts and cognitive capacities that are unique to humans and variable across cultures?



Two possible answers

1. There are other, uniquely human systems of core knowledge.
 - ex: a core system underlying communication and cultural learning (Tomasello; Gergely & Csibra)
 - a core system for reasoning about coalitions, cooperation and competition, social groups (Cosmides & Tooby; Dunbar)
2. There are uniquely human processes by which children go beyond the limits of core knowledge.
 - ex: natural language may serve to combine representations from different core domains, both flexibly and productively.

Case 1: Objects

Young children represent artifact objects as kinds, made for some purpose (Kelemen & Bloom).

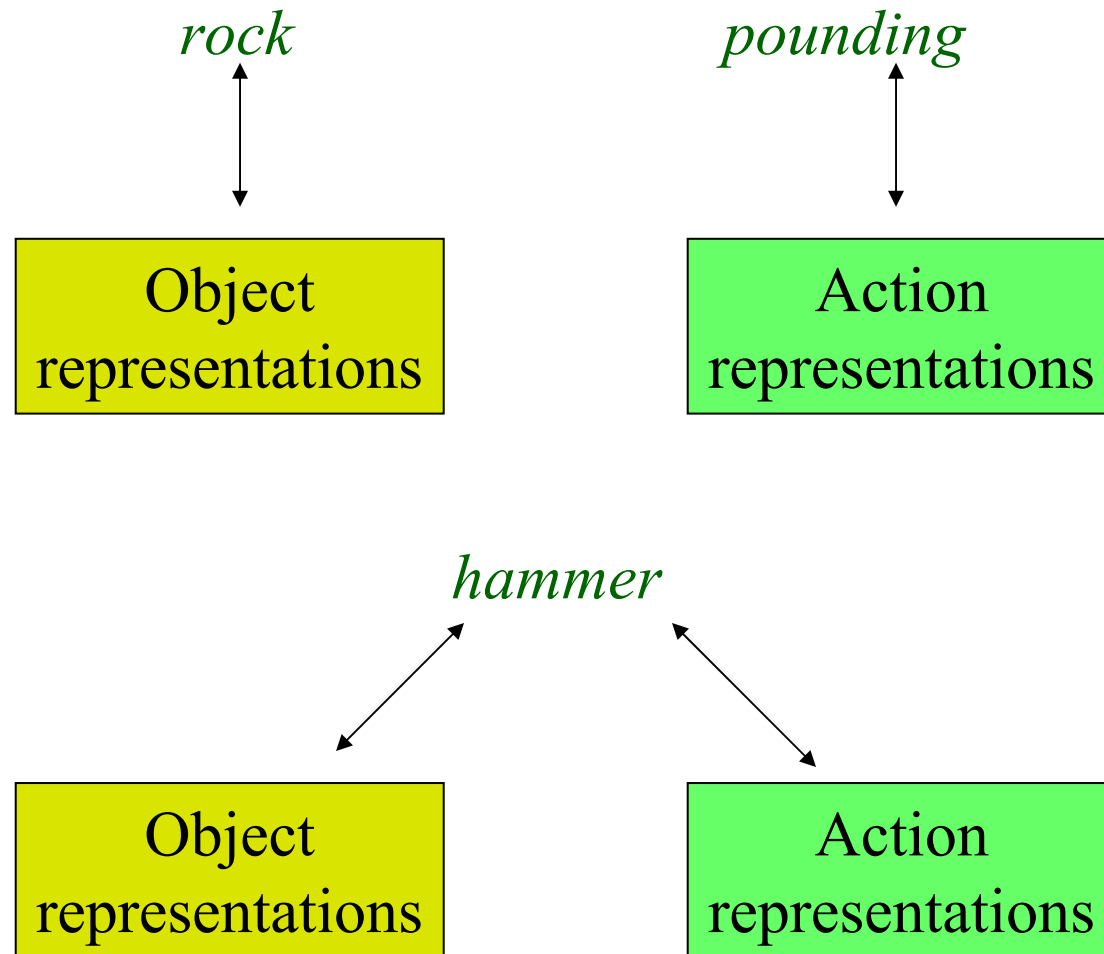
Non-human primates may not share these object representations.

Infants' object representations may change with the acquisition of natural language (Xu & Carey, 2001)



A suggestion: Uniquely human encyclopedic knowledge of object kinds and promiscuous tool use may depend in part on language.

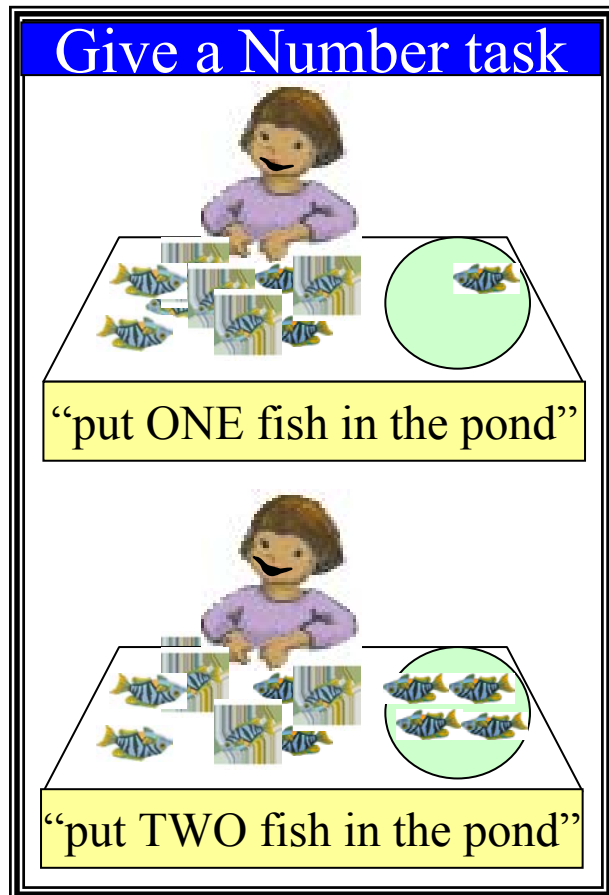
Beyond core knowledge of objects: Tools



Language may provide a medium for combining information about objects and actions to permit rapid and flexible tool use.

Case 2: Natural Number

Children go beyond the set size and ratio limits and begin to represent exact, large numbers when they learn verbal counting.



2.5 years: “one” = ONE
“two” etc. = SOME

3 years: “two” = TWO

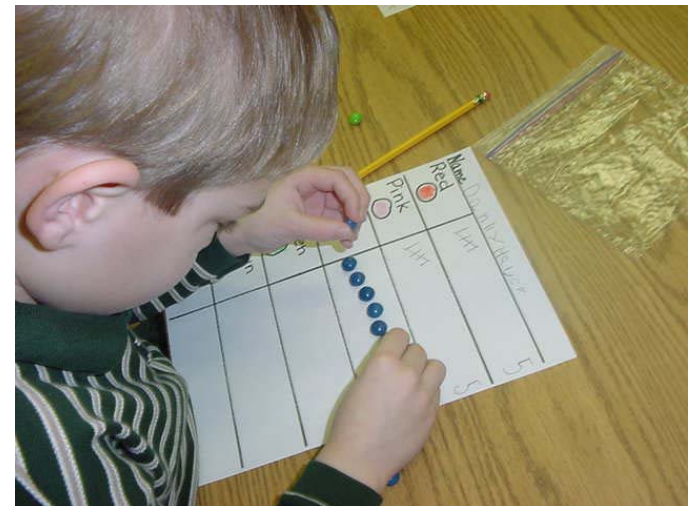
3.5 years: “three” = THREE

4 years: each word in count
sequence picks out a distinct,
exact number; counting serves
to determine that number

In preschool children, knowledge of natural number is productive:

Five-year-old children who cannot count beyond “sixty” understand that “eighty-six” picks out an exact cardinal value and no longer applies if an element is added to or removed from the set.

(Lipton & Spelke, 2006)



No evidence for this learning in non-human animals

chimpanzees: the case of Ai (Matsuzawa, 1985, 1998)

parrots: the case of Alex (Pepperberg, 1987)

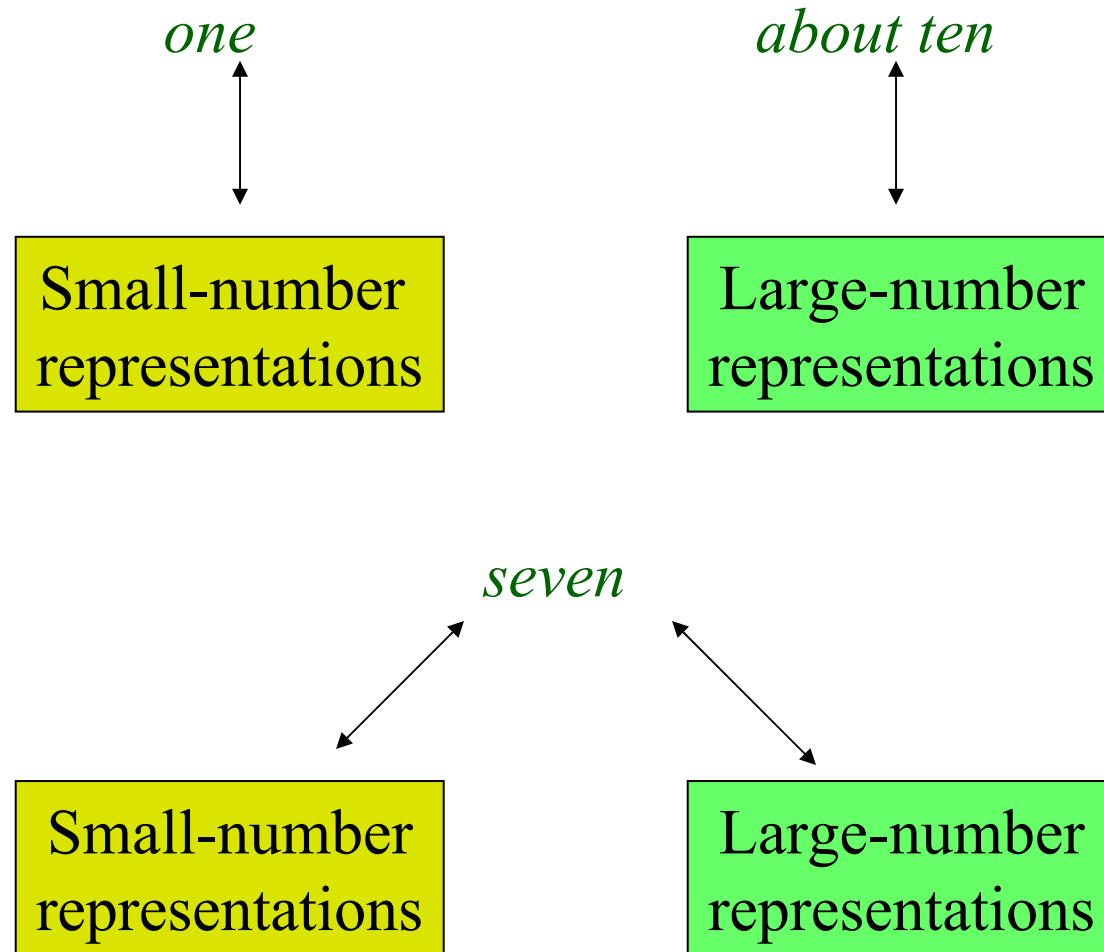


(Ai: 20 years of training, 1-9)



(Alex: 28 years of training, 1-6)

Beyond core knowledge of number



Language may provide a medium for combining information flexibly about small, exact and large, approximate numbers to permit the construction of natural number concepts.

Case 3: Space

Children combine geometric and landmark representations at about 6 years; this development correlates with the onset of spatial language (Hermer-Vazquez et al, 2001)

Children taught spatial language may show enhanced reorientation (Shusterman & Spelke, 2005)

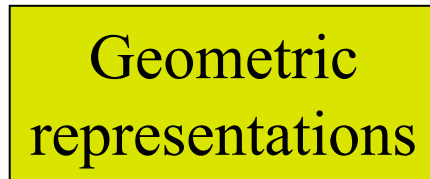
Adults given verbal interference show impaired reorientation (Hermer-Vazquez et al., 1999).



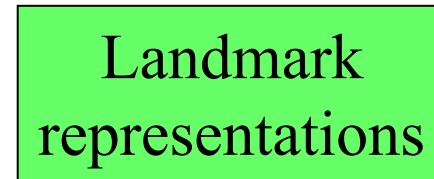
Uniquely human capacities to navigate flexibly, combining geometry and landmarks, may depend in part on language.

Beyond core knowledge of space

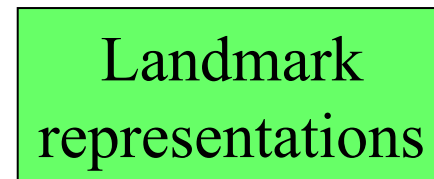
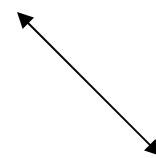
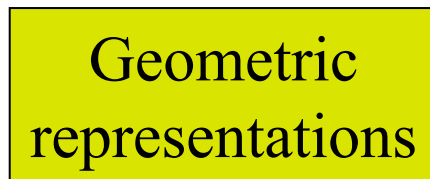
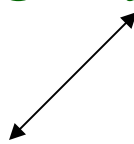
right of the valley



the red wall



right of the red wall



Language may provide a medium for combining information about geometry and landmarks rapidly and flexibly.

How can language support new representations of objects, space or number?

Languages consist of **words** and **rules**.

Words can be coined for entities in any conceptual domain:

wall, blue, long, left...

Words can refer to entities whose properties collect together information from multiple domains:

hammer, seven

Rules conjoin words irrespective of conceptual domain:

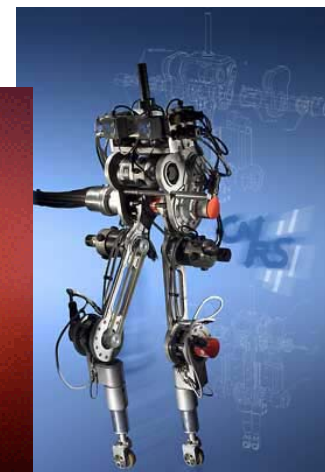
left of the blue wall

seven deadly sins

Language may provide a medium for combining information rapidly and productively, overcoming the limits of domain-specific, encapsulated core knowledge systems.

How do humans develop complex cognitive skills?

Some tools for finding out:



Thank You!

Katie Kinzler

Justin Wood



Kirsten
Condry

Sang
Ah
Lee

Ariel Grace

Anna Shusterman

Kristin Shutts

Aside: Are males better at representing small numbers of objects?



Claims: Males are biologically predisposed to greater interest in, and better processing of, objects.

--L. Summers (2005)

--S. Pinker (2002)

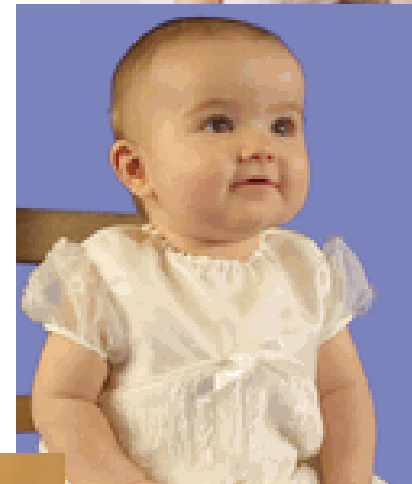
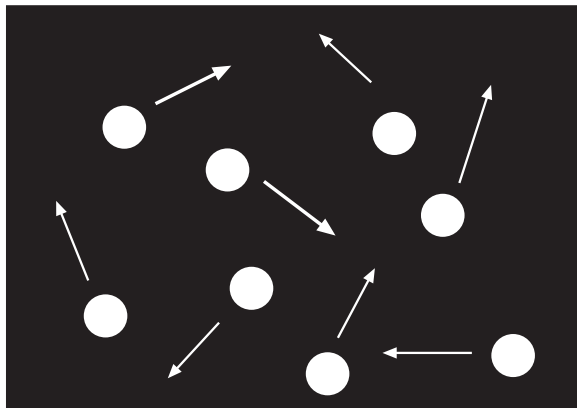
--S. Baron-Cohen (2003, 2005)

Aside: Are males better at representing objects?

Evidence: Most studies of infants show equal performance by male and female infants (Wynn, 1992; Feigenson et al., 2002). One study shows a female advantage (vanMarle, 2005).

But: yes-no tasks
small Ns, limited data

Multiple object tracking:



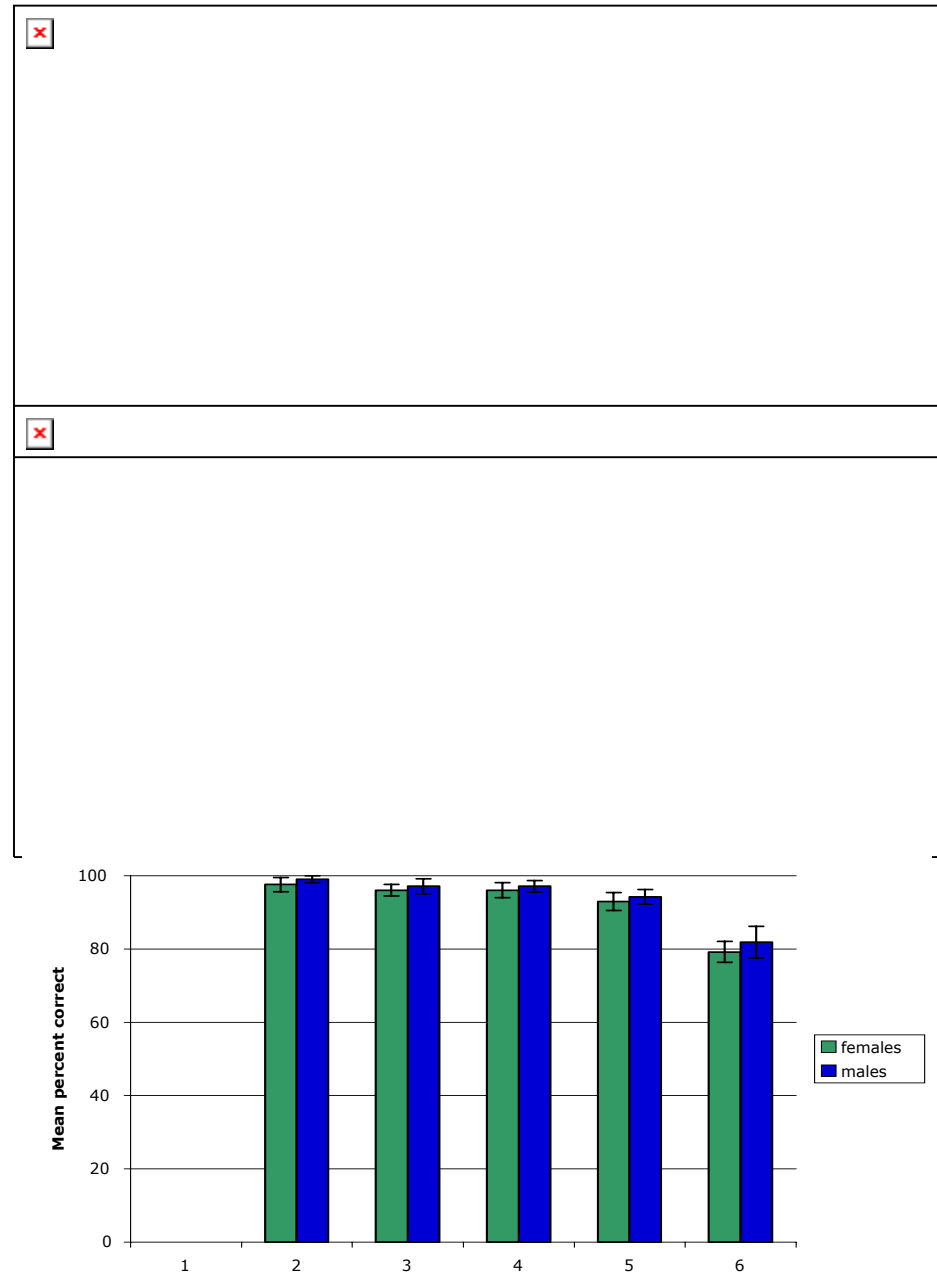
Sex differences in multiple object tracking? (N=104)

Children (4-8 years)

Children (9-13 years)

Adults

Similar performance in males and females.



Sex differences?



No significant differences; convergent performance by sex

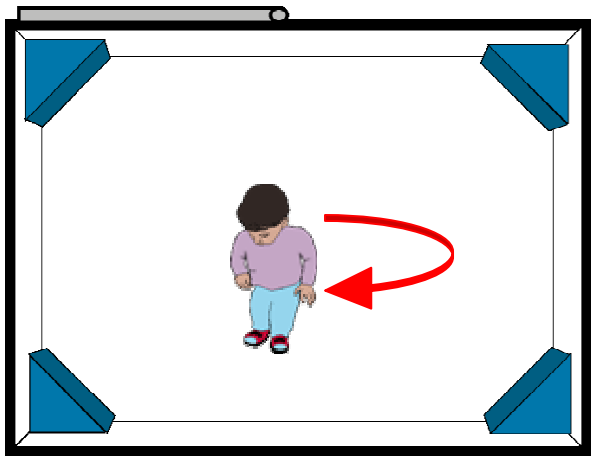
Another aside: Sex Differences?

Pinker, Geary, and others:

yes

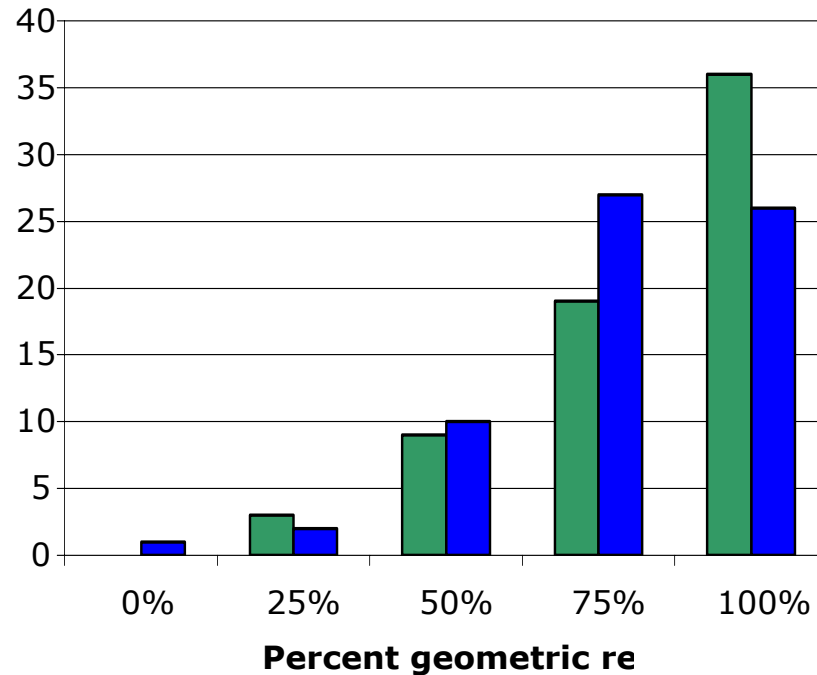


geometry/rotation



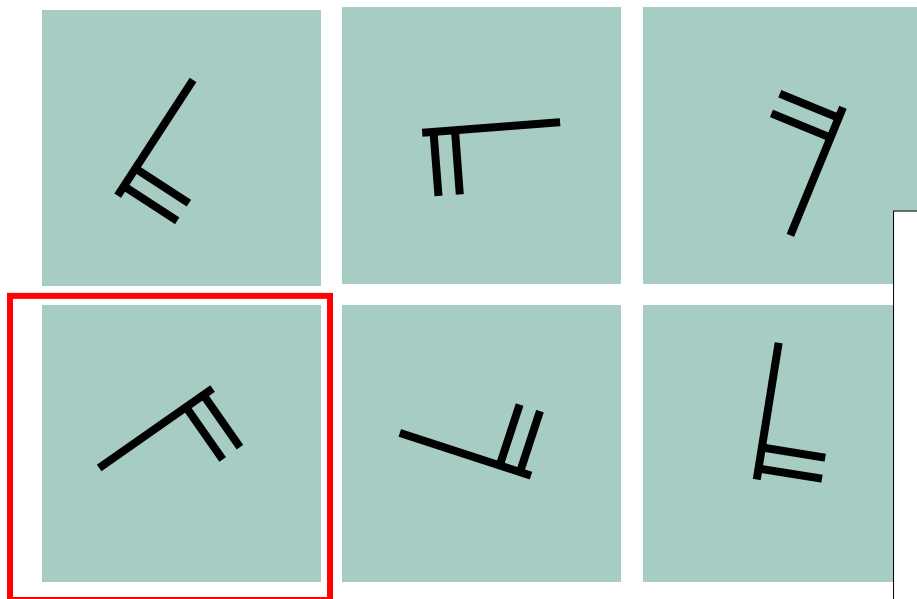
But: too easy?
too young?

Use of geometric info
(N = 133; 4.0-4.5 ye

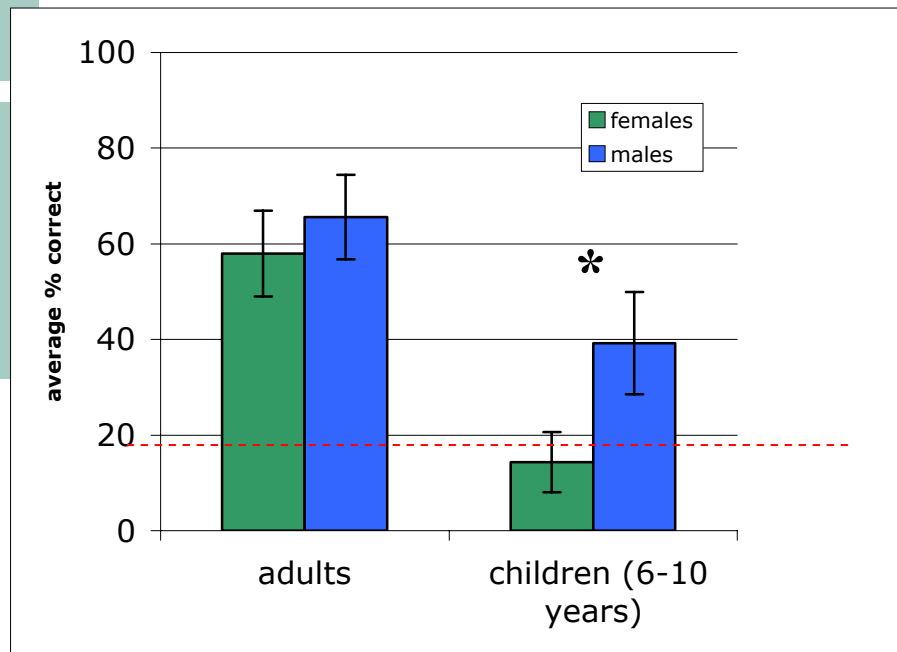


Shusterman, Lee & Spelke, aggregate data

Mental Rotation

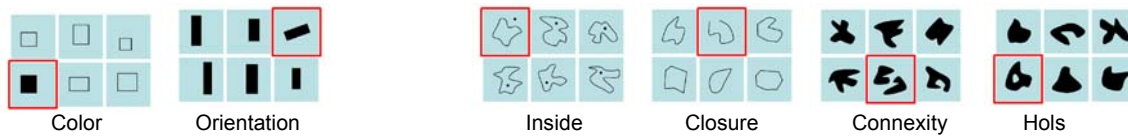


How general is this effect?

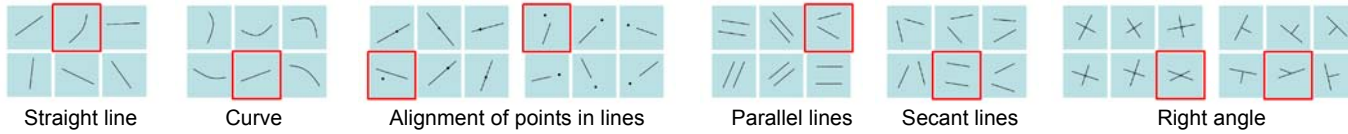


*p < .05

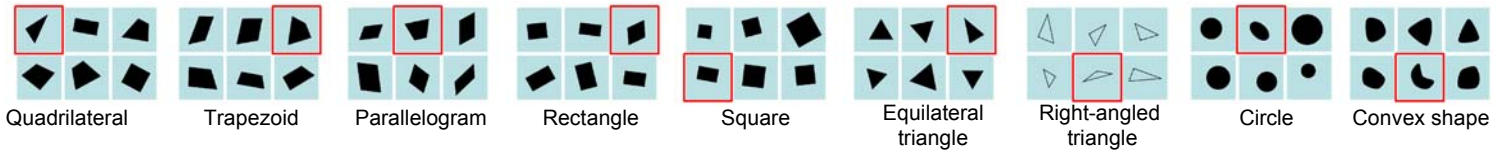
(Grace, Shutts & Spelke, in prep.)



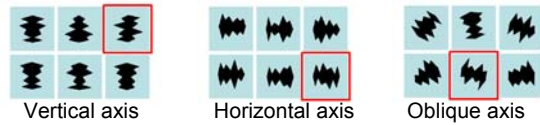
B Points and lines



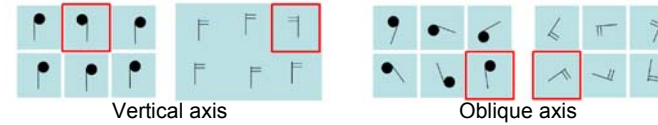
C Figures



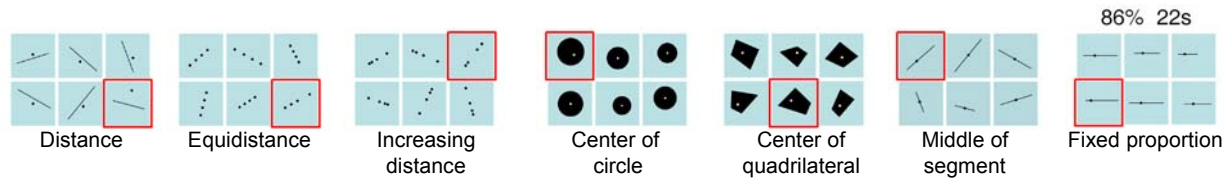
D Symmetry



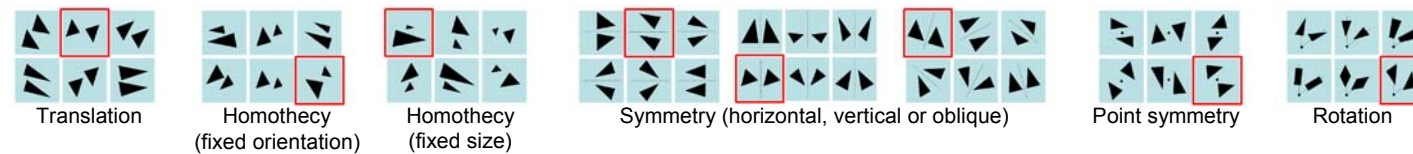
E Chirality



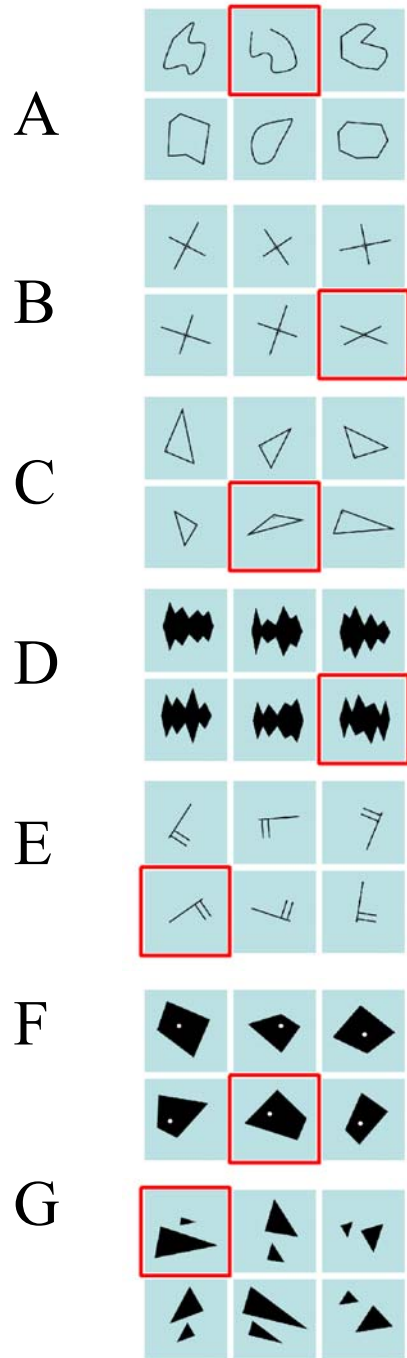
F Metric properties



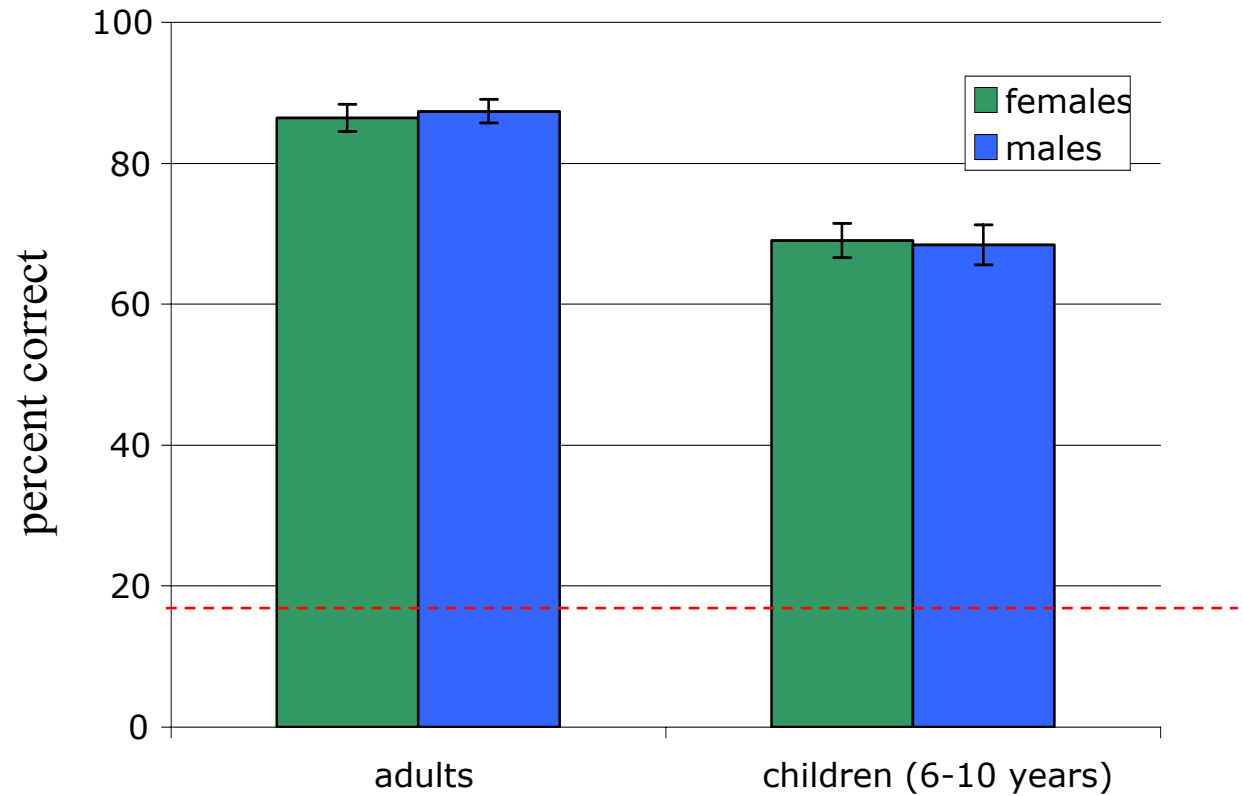
G Geometrical transformations



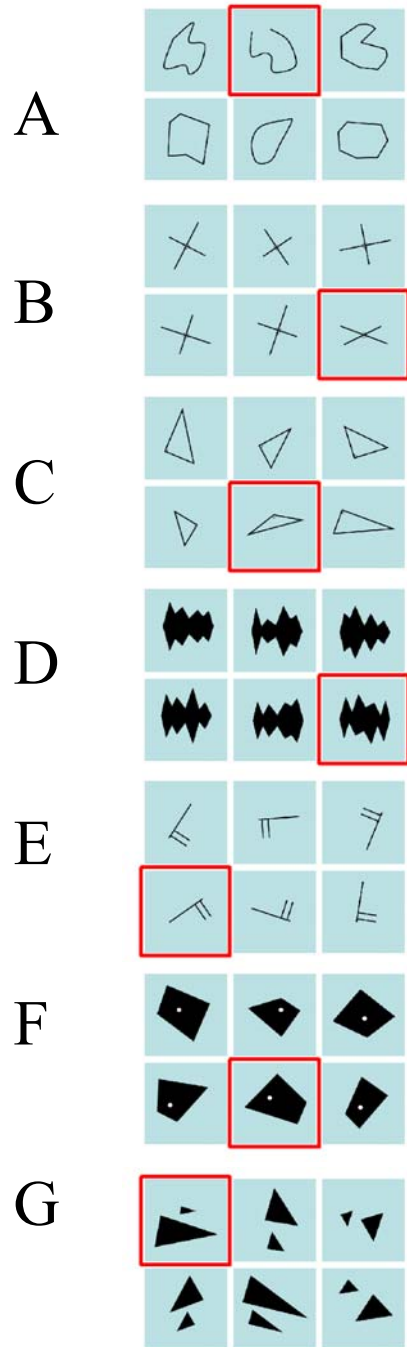
Sex Differences?



Overall Performance

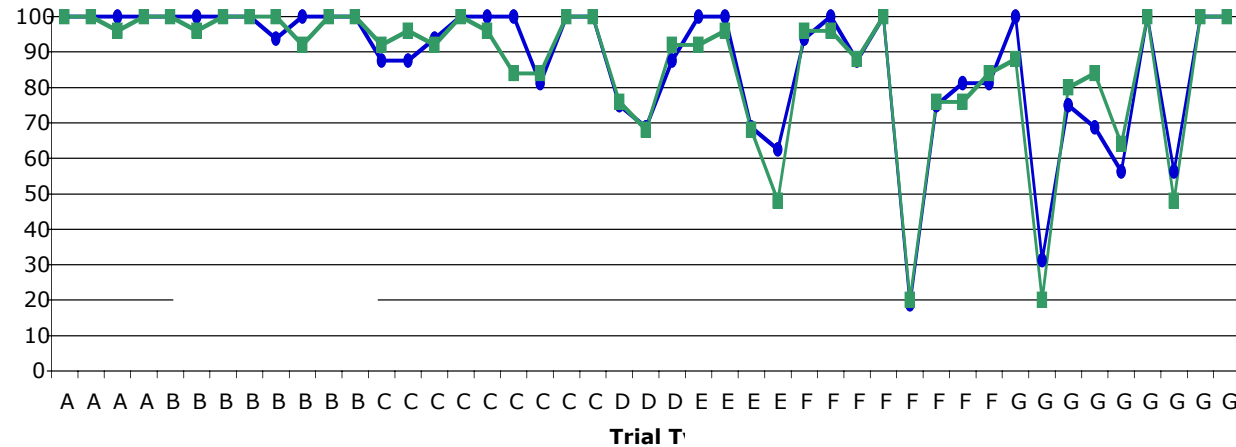


(Boston sample, n=90)

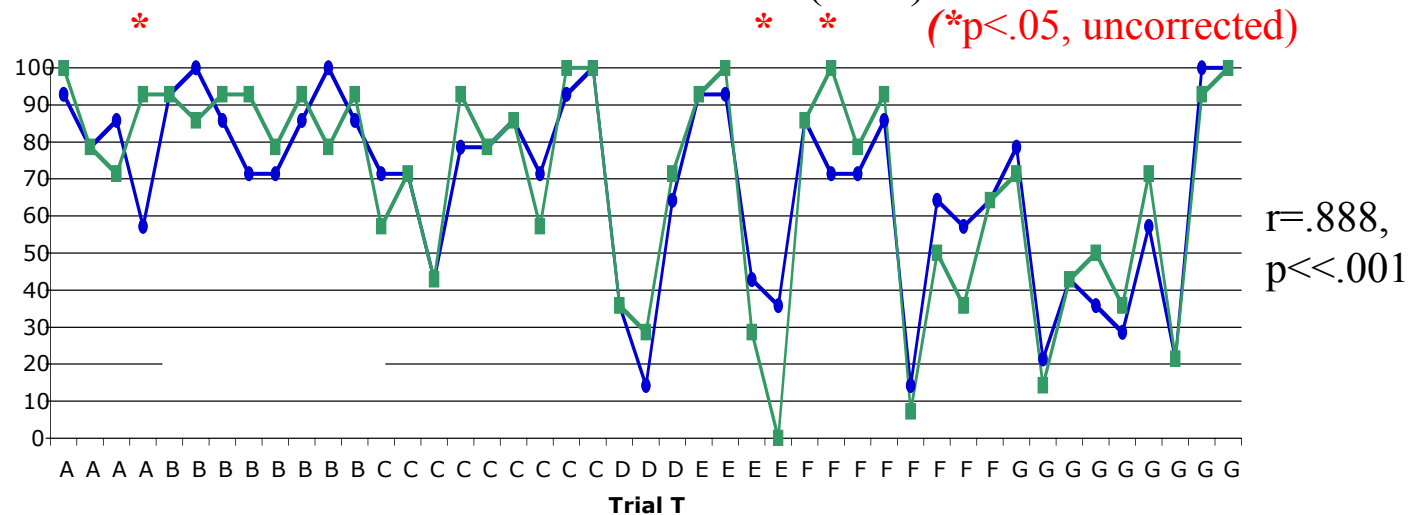


Geometry performance by item

Boston Adults

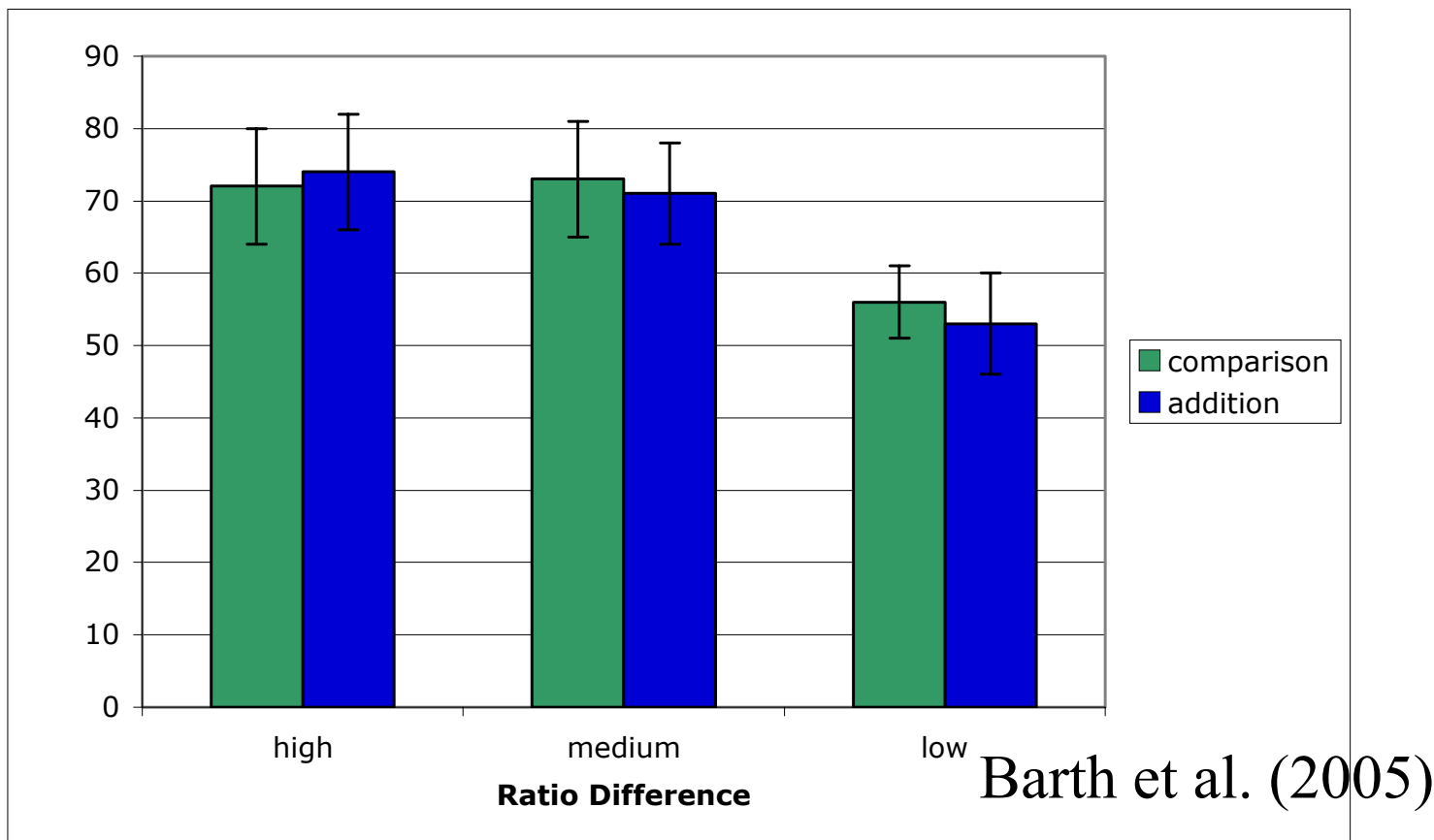


Boston Children (6-10)



Highly convergent performance of males and females.

Visual comparison and addition (5-year-old children)



Preschool children compare and add arrays of dots.
Performance depends on the set size ratio.
Addition is almost as accurate as comparison.